A Load of Balls?
Juggler's Slide Rules - System Tombeur

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Warning! This article describes frivolous devices which serve little practical purpose, they were designed and made as an academic exercise, but have also turned-out modern-day tributes to the versatility of the slide rule.

This is the story of the development of a pair of 21st century slide rules for a completely new application - juggling - and how to use their "System Tombeur" scales (Figure 1).

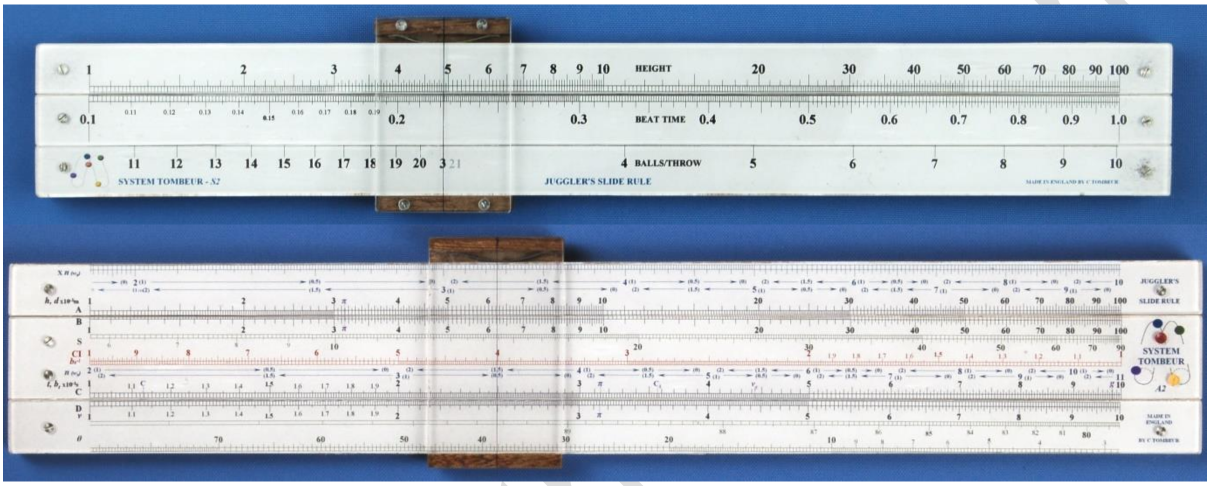


Figure 1: Finished articles, the Simple Juggler's Slide Rule (top), and Advanced Juggler's Slide Rule (bottom).

Juggling and Slide Rules

Firstly, yes, it is possible to juggle slide rules, the easiest way being by throwing them with end over end spin as one would when juggling with clubs, but that is not what this endeavour was all about.

Juggling is a physical skill involving the manipulation of objects for recreation, entertainment, or sport. It can involve one or many objects at the same time, using one or many hands, and has been in recorded history for 4000 years. The most recognizable form of juggling is toss juggling where props, usually balls or clubs and outnumbering the hands used, are woven into beguiling patterns with throws and catches such that at least one object is always in the air. Juggling is often described as an art, it can be performed anywhere, and everyone can learn to do it. It can be simple and concise or complicated and confounding. When done well it is mesmerising and beautiful to both perform and watch.

Slide rules were invented in the early 1600's and became the prevalent personal mechanical calculating aid until the mid-1970's, when they quickly became obsolete thanks to the rapid development of the electronic calculator. There were many manufacturers, and many different designs of varying complexity, with a range of applications from general calculation to specific purposes in science, engineering, and commerce. However, to my knowledge, during their 350-year prevalence, there was never a slide rule designed specifically for calculations relating to juggling. And probably for good reason, but more on that later.

Me, Juggling and Slide Rules

Juggling was something that interested me, and I felt I ought to be able to do, long before I eventually got round to mastering (at least to some degree). Having learnt the basics with balls, I progressed with different patterns and techniques, added a couple more balls and learned how to juggle with clubs. Being an analytical type and interested in maths and physics, I then became interested in the mechanics of juggling, that is how the patterns work; how it all fits together in space and time.

Until 2011, several years after I got the juggling bug, I had never used a slide rule, in fact I had never even touched one. I missed them at school by a few years, where it was all log tables and then electronic calculators for me. I was aware of the existence of slide rules, but in my ignorance, I considered them to be complicated, obsolete curiosities that I would get round to investigating at some point, probably. That point happened by chance when I decided to investigate a filthy, unidentified object in a box of tools given to me by my father. The object turned out to be a very old alcohol slide rule. My eyes were opened, and I promptly became fascinated by these tactile, elegantly ingenious, and yet blinding simple devices.

It struck me quite quickly that the visual, hands-on art of juggling could in some way be a great application for such a visual and hands-on tool. Initially I thought that I could make a mechanical tool that would help in designing new juggling patterns. I quickly dismissed this idea as impractical, but the more I delved into understanding how slide rules worked, the more I realised that juggling would be a great application for a calculating rule. Juggling patterns are predictable, governed by the laws of physics, and simply described mathematically with equations that could be realised on a slide rule so that all manner of data and relationships could be calculated and explored. Juggling and slide rules were about to cross paths for the first time after 400 years with the invention of a slide rule to describe simple concepts of the mechanics of juggling. Admittedly this was a bit late to a party that had in fact finished three and a half decades previously!

First, I satisfied myself that such a thing did not already exist, then I set about designing and making one. Thus the "Juggling Slide Rule", as I first called it, was born. In fact, two versions were born, a "Simple" model for the basics and a "Complex" model for more advanced concepts (Figure 2). Both models were for calculations relating to "toss juggling" (throwing and catching objects), but specifically where spin does not affect the mechanics, for example juggling with balls rather than clubs or rings, where the spin does affect the flights of the objects.

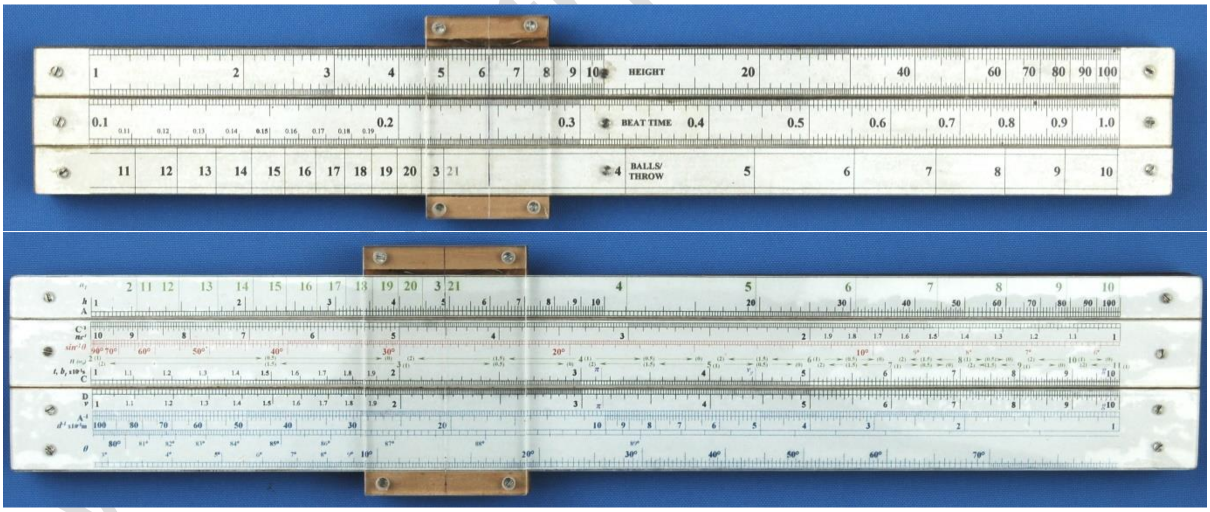


Figure 2: First efforts, Simple Juggling Slide Rule (top) and Complex Juggling Slide Rule (bottom)

In some respects, my Juggler's Slide Rules are cousins of artillery slide rules. Both types are concerned with ballistics, but whereas an artillery slide rule is concerned with forces required for trajectories on a large scale, the juggler's slide rule is concerned with the timing of small interlocking trajectories where atmospheric conditions and terrain etc are not a factor.

My background in maths, science and computing meant the theory and subsequent programming to computer generate the scales was not too problematic. I also enjoyed physically making things, so I resolved to make my new slide rules look and function as professionally as possible within the bounds of my limited skills and facilities. The resulting slide rules are single sided, closed frame devices, made from laminated hardwood strips, with printed paper scales, more-or-less standard 25cm scale length, faced with acrylic, with a sprung acrylic and wood cursor. This simple description belies a lengthy development process from conception to finished article, that proved to be far more challenging than I expected, throwing up many expected, but even more unexpected challenges. My eyes were opened to complicated design and production concepts that I previously had little appreciation of, such as designing and producing practical layouts, legibility, tick mark frequencies, materials, construction, writing comprehensive instructions that are also compact and easy to follow, and even just explaining the complexities of juggling to the layman. This design and development process is beyond the scope of this article but may form the basis of a future one.

An Academic Exercise

Here I must address the warning at the very top of this article, and its playful title. Most practicing jugglers are not concerned with the maths or physics of juggling - they can juggle (timing, placement etc), it works, and precise analysis is not relevant. In fact, they are juggling by feel, intuition and experience gained from *a lot* of practice. Jugglers generally do not think about the mechanics either when they learn, practice, or try new things. Some, may consider the mechanics when designing new patterns, but this had more to do with how all the objects and hand moves fit together in the pattern, not the quantifications of heights and timings etc. My slide rules do calculate these things, and so they are of little to no use to the performing juggler for whom they were designed, at best they are an interesting aside. And, ironically, these slide rules cannot actually describe the juggling mechanics involved in actually juggling them!

I wholeheartedly acknowledge the above facts and appreciate the irony. Yes, these slide rules may be a frivolous, even pointless academic exercise, but I designed and built them because I was interested, and I have shared my designs because others have shown an interest.

Re-development into Juggler’s Slide Rule

I became a member of the United Kingdom Slide Rule Circle (UKSRC) around the same time that I was designing and making the Juggling Slide Rules, and I showed my completed creations at one of the local meetings. Those present showed interest in what I had achieved, mainly I suspect because of their novelty - a new application and design of an old technology did not come along very often! I was invited to give a short demonstration at the following years International Meeting of Slide Rule Collectors at Bletchley Park in 2012, which I duly did. Once again there was interest from those attending, with some expressing an in interest in acquiring one of my creations.

By then, however, the designs were over a year old, and on reflection there were several aspects of them that I was not happy with. The original scale designs were cluttered and overcomplicated, perhaps even partly deliberately in my naivety. So, I decided that before I made any for interested parties, I would give them an overhaul. My collaborator in this endeavour was David Rance, who was very enthusiastic about the concept, and a willing partner for me to bounce my ideas off. David actively encouraged my improvements to the design/functionality and made many suggestions including adding professional touches such as naming the scale layout, writing detailed instructions and dating/serial numbering all the slide rules that I made.

The redesigned slide rules are those shown in Figure 1. Once again there are two more-or-less-standard size versions, a “Simple” version (S2) for the basics, and an “Advanced” version (A2, renamed from “Complex”) which can also be used for more sophisticated juggling concepts. There is an additional Simple model, the S3, which is a half-size “pocket” version of the S2, another of David’s great ideas. The construction is almost the same as the first models, but with some small improvements learned along the way.

I renamed the slide rules “Juggler’s Slide Rules” to avoid ambiguity about actually juggling *with* slide rules, and added a small juggling pattern logo (actually a representation of a five-ball cascade). The design of the scales/layout for the juggling calculations was named “System Tombeur”, thanks again to David who knew that I am interested in early Faber-Castell slide rules. He suggested this idea in homage since Faber named some of their novel slide rule scale systems after their inventors.

The Advanced rule scales are now also much more slide-rule-user friendly than the previous Complex model. It has the standard Mannheim scales (A, B, CI, C and D, plus Sin scale) for normal calculations, which are also utilised for some juggling calculations, with the addition of specific some specific juggling scales. The old asymmetric look of the Complex rule, where the bottom rail of the stock was wider than the top rail, which I was never happy with, is also gone on the new Advanced rule.

As previously, summary instructions and methods for determining magnitudes are printed on the back of the stocks, but with the addition of a blind stamped production date (*MM* & *YYYY*) and serial number (*nnn*) on the left and right respectively (Figure 3).

Figure 3: Reverse side, the Simple Juggler's Slide Rule (top), and Advanced Juggler's Slide Rule (bottom).

Juggling Basics - Pattern Speed, Height & Throw Weight (Simple and Advanced Models)

There are two fundamentals to most juggling - the pattern and gravity. The pattern is a repeating sequence of throws of objects that lock together, and gravity determines the flights of the objects and hence the speed of the pattern. Gravity is the juggler's greatest enemy, a foe they are ultimately doomed to fail against, but it is also their greatest ally because they could not juggle without it. To understand how juggling can be applied to a slide rule, it is first necessary to know a little about how juggling patterns work.

Common juggling patterns usually consist of a number of balls all thrown to the same height, or a sequence of throws of different heights which lock into a repeating pattern. Throws are made from two hands alternately and regularly to a beat. The time between each beat in seconds is known as the **Beat Time** and determines the speed of the pattern. Pattern speed can also be expressed as number of throws per second which is the inverse of the Beat Time. The **Weight** of the throw of a ball, measured in beats, indicates when that ball will be next thrown in the pattern. In patterns where all the balls in the pattern are consistently thrown the same, the weight of each throw is the number of balls in the pattern. Patterns like this that are based on an odd number of balls are known as 'cascades', whereas those based on an even number of balls are known as 'fountains'.

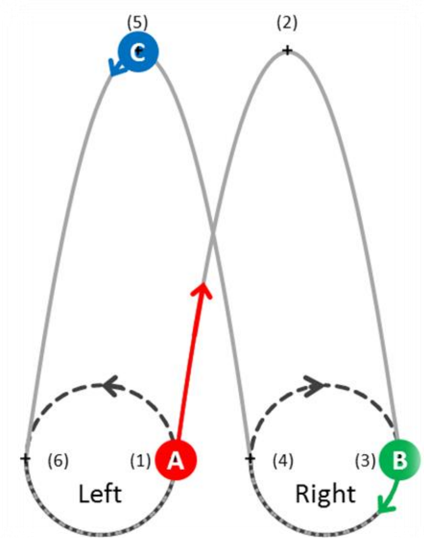


Figure 4: 3-ball cascade (beats in brackets show position of ball A).

For example, in a 3-ball cascade, the starting point for most jugglers, each hand throws a ball alternately once per beat and all the balls are thrown are the same (Figure 4). Since there are three balls, then each ball is next thrown three beats later, so the weight of each throw is called a 'three' (or '3'). Similarly, in a 4-ball fountain, all throws are the same, so the weight of each throw is a '4'.

The pattern '534', has four balls thrown with the repeating weights '5' then '3' then '4'. More complex patterns like this are usually named with the repeating part of the sequence, in this case '534' from throwing 534534534534…

In a theoretically neat pattern, the hands trace small circles at a constant rate, as seen from the juggler's perspective in Figure 4. When a ball is thrown by one hand another ball is caught by the other hand. Each hand is full for one beat and empty for one beat. The length of time a ball is held in the hand (between being caught and thrown) is called its **Hold Time**, and is measured in beats.

The weight of each throw determines the airtime in beats of the ball thrown as its weight minus one, since each ball is held in a hand for one beat. The flight of the ball is predictable according to the laws of projectile motion, and its maximum **height** can be determined from its airtime. Hence for different throw rates, heights for different weights can be calculated or compared. Similarly, throw rates for different weights and heights can be calculated or compared.

It is often acknowledged that a juggler has learned a pattern when they can accomplish two complete cycles of throws and catches, for example six successful throws and six successful catches of a three-ball cascade. At this stage, the pattern is likely to be jerky and chaotic, but with practice it will become a smooth, consistent, and seemingly effortless approximation of the theoretically neat shape shown in Figure 4. It may take an afternoon to 'learn' to juggle a 3-ball cascade, a few weeks for a 4-ball fountain, and many, many months for a 5-ball cascade, considered a juggling landmark that few go beyond in terms of number of balls juggled.

The Simple Juggler's Slide Rules - Layout and Scales

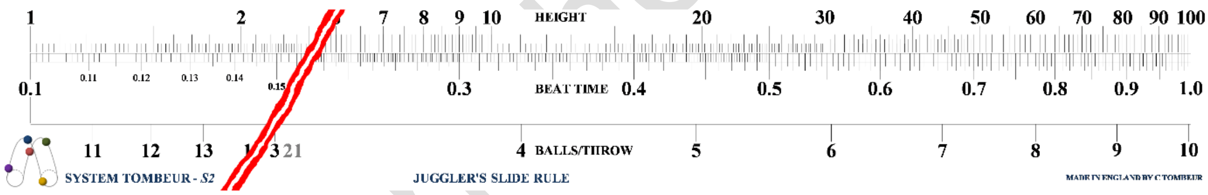


Figure 5: Simple Jugglers Slide Rule scales (shown abridged)

Simple Juggler's Slide Rules enable quick calculation and comparison of heights and speeds of common ball juggling patterns where two hands throw alternately to a regular beat of time. The scales (Figure 5) are optimised for theoretically neat patterns where each hand is full (between catching and throwing) for the same length of time that it is empty (between throwing and catching). The single sided slide rule has 3 scales - one on each of the top and bottom front faces of the stock, and the third on the slide:

- 'HEIGHT' scale, 1-100, on the top of the stock can represent centimetres or metres.
- 'BEAT' scale, 0.1-1 second, is in the middle on the slide.
- 'THROW' scale, 11-21 and 3 -10, on the bottom of the stock is graduated for throw weights (or number of balls in a pattern where all throws are the same) of 3 to 21, with values 3 and 21 sharing the same tick/graduation mark.

The Height scale is a standard logarithmic slide rule A scale, and the Beat scale is a standard logarithmic slide rule C scale but renumbered to represent seconds from 0.1 to 1. The Throw scale is a scale of gauge marks, actually a renumbered and offset standard logarithmic slide rule D scale. A cursor with single hairline aids calculations and comparisons.

Using the Simple Juggler's Slide Rule for Basic Juggling Calculations and Comparisons

To determine the maximum height achieved by a ball thrown at a certain weight and at a particular beat time in seconds, first set either the left or right-hand index of the slide (0.1 or 1 mark) to the throw weight mark on the THROW scale. Find the beat time on the BEAT scale on the slide, and then read the height directly against it on the HEIGHT scale. If the desired beat time falls outside of the HEIGHT scale use the index on the other end of slide. Magnitudes for height can be determined as follows:

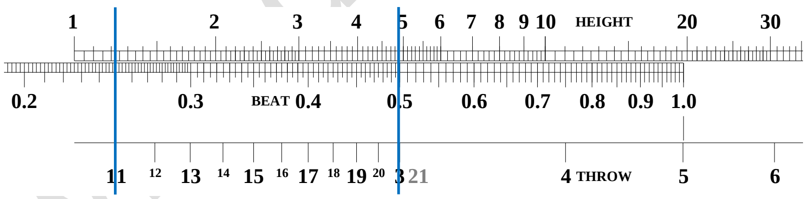
- For throw weights *up to 10*, and when the slide protrudes from the stock to the *right*, then the height is in *centimetres*.
- For throw weights *up to 10*, and when the slide protrudes from the stock to the *left*, then the height is in *metres*.
- For throw weights *11 and above*, and when the slide protrudes from the stock to the *right*, then the height is in *metres*.
- For throw weights *11 and above*, and when the slide protrudes from the stock to the *left*, then the height is in *metres times 100*.

Once an index on the slide is set to a weight mark on the THROW scale, corresponding heights can be read for all values on the BEAT scale that are in the HEIGHT scale range. Setting the other index to the weight enables heights to be read against beat times that were previously off the scale.

Similarly, beat times can be read on the BEAT scale for specific heights on the HEIGHT scale, and setting the correct index on the slide to the weight mark on the THROW scale will ensure that values can always be read.

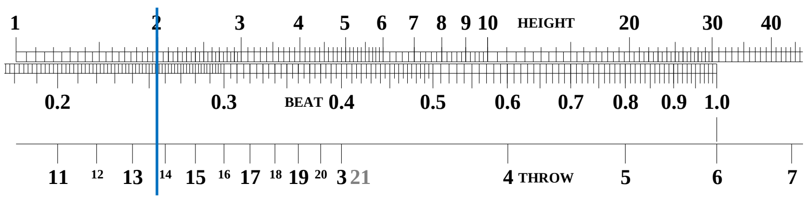
The graduations on the BEAT scale can also be used for beat times from 1 to 10 seconds rather than 0.1 to 1 seconds. Height magnitudes are then determined by using the rules above, and factoring again by 100 (since height is proportional to the square of time).

Example 1: To what respective heights would the balls in a 5-ball cascade have to be thrown to achieve a throwing beat or rhythm of 0.25 seconds or 0.5 seconds?



*Set the right-hand index of the BEAT scale against "5" on the bottom THROW scale. Next position the cursor hairline over "0.25" on the BEAT scale. Using the hairline the corresponding throw height of **1.23 metres** can now be read off on the top HEIGHT scale. Simply repositioning the cursor hairline over "0.5" on the BEAT scale gives the alternative throw height of **4.91 metres** on the HEIGHT scale.*

Example 2: What throwing beat or rhythm is needed to sustain a 6-ball fountain when the balls are juggled/thrown to a height of 2 metres?



*Set the right-hand index of the BEAT scale against "6" on the bottom THROW scale. This time place the cursor hairline over "2" on the top HEIGHT scale. The corresponding throwing beat or rhythm of **0.253 seconds** is under the hairline on the middle BEAT scale.*

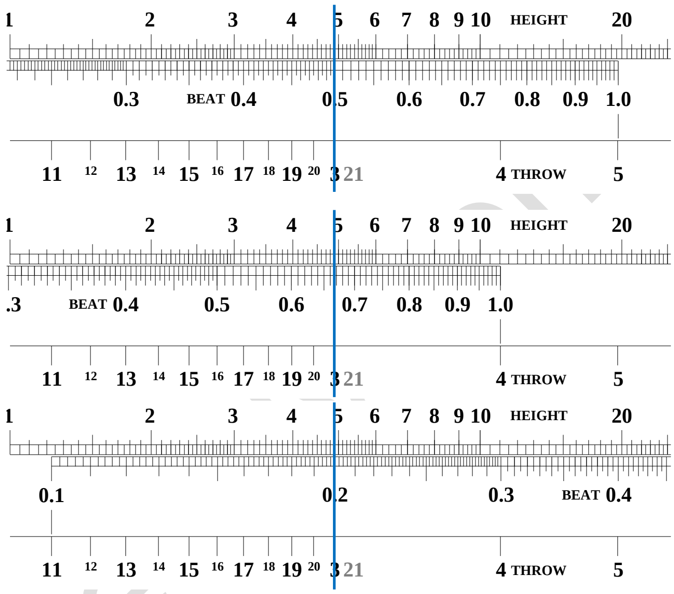
To find the beat times for different throw weights to the same height, position the cursor hairline over the height on the HEIGHT scale (which could be the result of a previous calculation) and move the left or right-hand index of the slide to different throw weights marks on the THROW scale. For each throw weight set, the cursor hairline gives the corresponding beat time in seconds on the BEAT scale.

Example 3: The height of a 5-ball cascade thrown to a beat of 0.5 seconds is 4.91 metres. What would the rhythm need to be to maintain this height with a 4-ball fountain and an 11-ball cascade?

As before, set the right-hand index of the BEAT scale against "5" on the bottom THROW scale and position the cursor hairline over "0.5" on the BEAT scale.

Now move the slide so that the right-hand index of the BEAT scale is against the "4" on the THROW scale. Read the corresponding rhythm of ***0.667 seconds*** *on the middle BEAT scale under the hairline.*

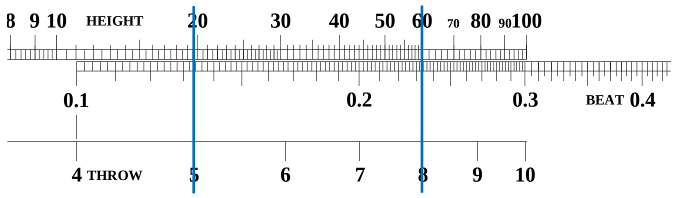
Re-position the slide so the left-hand index of the BEAT scale is against the "11" on the THROW scale. Again read the rhythm on the BEAT scale under the hairline of ***0.2 seconds****.*



Multiples of beat time or throw height for different throw weights can easily be determined. Set the left or right-hand index on the slide to the lower throw weight mark on the THROW scale. Read the value on the BEAT scale against any higher throw weight mark. This value (ignoring the decimal) is the beat time multiple when these two different throw weights are to the same height. Set the left-hand index on the slide to 1 on the HEIGHT scale, read the value on the 'HEIGHT scale against the beat time multiple on the BEAT scale. This value is the height multiple for these two throw weights when they are thrown to the same beat time.

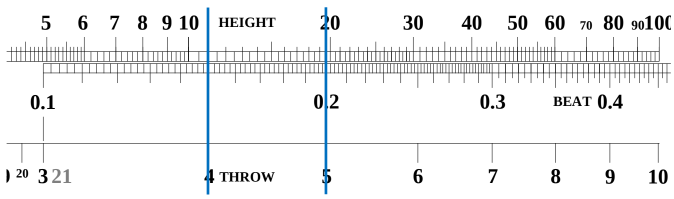
Example 4: How many times faster than a 4-ball fountain would a 5-ball cascade and 8-ball fountain need to be juggled if their heights were all kept the same?

Set the left-hand index of the BEAT scale against the "4" on the THROW scale. Place the cursor hairline over the "5" on the THROW scale and read ***1.33 times*** *under the hairline on the BEAT scale. Now move the cursor hairline over the "6" on the THROW scale and read* ***2.33 times*** *on the BEAT scale under the cursor hairline.*

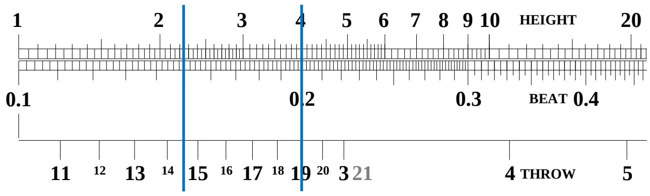


Example 5: In the pattern '534', how many times higher than the '3' must the '4' and '5' be thrown respectively?

First set the left-hand index of the BEAT scale against "3" on the THROW scale. Using the cursor note the values 1.5 and 2 on the BEAT scale corresponding to the "4" and "5" marks on the THROW scale.



*Then set the left-hand index on the BEAT scale against the “1” on the HEIGHT scale. Move the cursor hairline over 1.5 and then 2 on the BEAT scale and read the corresponding results of **2.25 times** and **4 times** higher on the HEIGHT scale.*



Advanced Juggling Concepts (Advanced Model Only)

To use and appreciate the Advanced model slide rule it is necessary to expand upon certain aspects of basic juggling. ‘Hold Time’ describes this concept in greater detail and how it affects pattern speed and height. ‘Throw Angle & Object Velocity’ introduces these characteristics and how they can be determined.

<u>Hold Time</u>

In theory the smoothest patterns are achieved when juggling with hold times of one beat. This is called the Base of the weight. In practice juggling with one beat hold times does not normally produce a smooth comfortable rhythm for the juggler. This is usually achieved with hold times of around 1.5 beats, depending on the pattern. When juggling with 2 hands throwing alternately to a regular beat, it is theoretically possible to have hold times of anywhere between 0 and 2 beats.

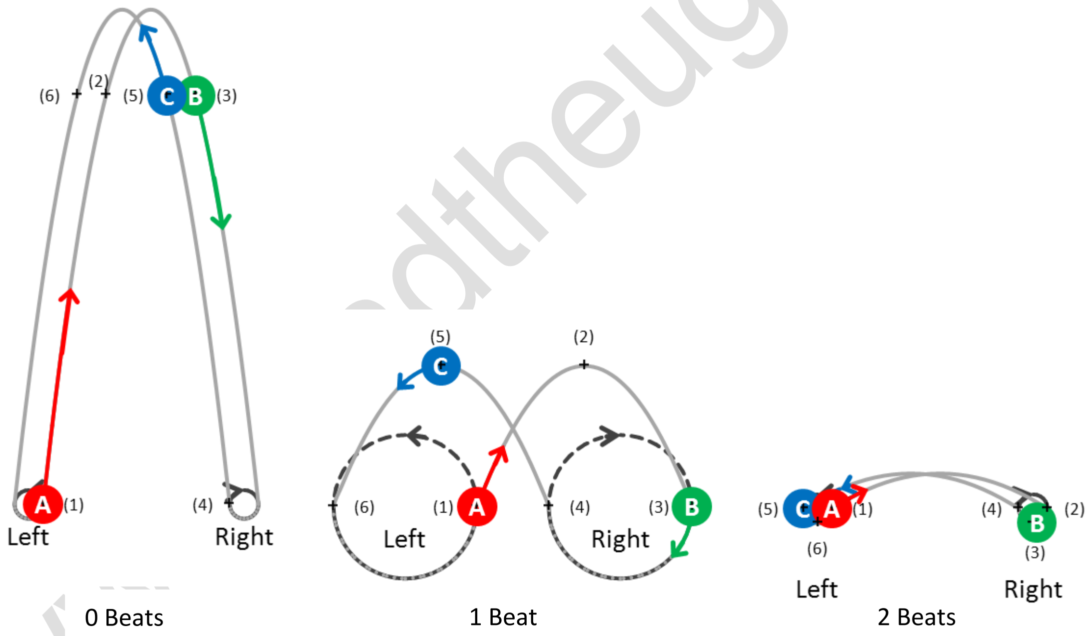


Figure 6: 3-ball cascade with 0, 1 & 2 beat hold times, from juggler’s perspective (beats in brackets show position of ball A).

Figure 6 shows a 3-ball cascade (each throw is a weight 3) with hold times of approaching 0 beats, 1 beat and nearly 2 beats. In each case, each ball is still thrown every third beat and each hand throws every other beat. With a hold time of one beat each ball is in the air for two beats, and each hand is full for one beat and empty for one beat. In the case of zero beat hold time, each ball is thrown as soon as it is caught, so the hands are almost always empty, and each ball is in the air for all three beats. When the pattern has a two-beat hold time, each hand only throws a ball when it needs to catch its next ball. In this case the hands are almost always full, and each ball is in the air for only one beat. It follows then that hold time for a throw weight cannot go below zero beats or over 2 beats in regular patterns because, respectively, either balls would never be in the hand or hands would have to manipulate more than one ball at a time.

The balls start and finish their flight at the same level, so if throws of the same weight and to the same beat are made with different hold times, then the maximum heights of the throws will be different because their airtime is different. Shorter hold times mean longer airtime, which mean higher throws. Longer hold times mean shorter airtime and therefore lower throws. So, a weight can have a range of heights for the same beat time, thanks to the hold time. Counter-intuitively, holding onto the balls for longer appears to produce a faster and more frantic pattern at the same beat because the airtime is less and the pattern lower.

Jugglers use different hold times in a pattern to create pattern distortions, which then become new patterns. A simple example of this is an 'over the top' throw, thrown during a 3-ball cascade. The throw is still a weight 3 thrown to the same beat as the rest of the pattern, but the ball is thrown over the top of the other balls. To achieve this extra height, the hold time of the ball is reduced and hence its airtime increased.

A more extreme example of this is the 3-ball pattern 'Arches' (Figure 7), which is a shape distortion of the 3-ball reverse cascade. A reverse cascade is simply a cascade where all the throws are made from the outside of the pattern towards the centre, the opposite direction to a normal cascade. In 'Arches' the first ball is thrown low, the second ball is thrown 'over the top' of the first, and the third ball is thrown 'over the top' of the second. This pattern then repeats, and a steady beat maintained. Each of the 3 balls is thrown to a distinctly different ascending height, to great effect, by shortening the hold times within the limits.

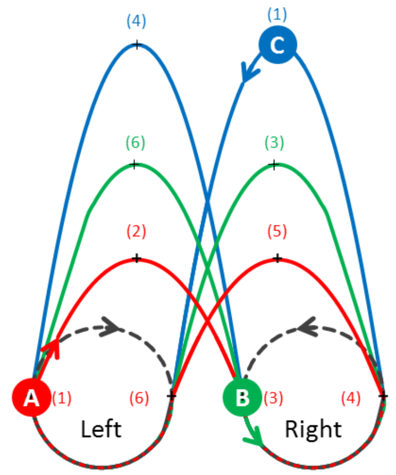


Figure 7: "Arches" (coloured beat numbers in brackets indicate ball position).

Many other complex patterns that involve throws around the body, different hand positions, crossing hands, under or overhand throws, differing throw heights or trajectories etc, are often just shape distortions of standard cascades or fountains. The extra time required to fit the moves into the regular pattern is gained by manipulating the hold times. Note again that the juggler does not necessarily appreciate this as they are juggling by feel. All these patterns, along with more or different objects, are often the next challenge for a juggler who has mastered a basic smooth 3-ball cascade.

Not only does hold time more accurately describe actual juggling, but hold times are a powerful tool for the juggler and they can easily be incorporated into calculations and comparisons of pattern speed, height and throw weight.

<u>Throw Angle & Object Velocity</u>

When an object is launched into the air, the laws of projectile motion dictate that due to the Earth's gravity the vertical speed of the object will decrease to zero, and then the object will accelerate towards the Earth. The rate of deceleration and acceleration due to gravity is a known fixed value at the Earth's surface, regardless of the weight or size of the object. In a normal juggling environment, where the props are relatively small, slow moving and unaffected by wind, air resistance is negligible and can be ignored. Gravity has no effect on the horizontal speed of an object.

Usually when a juggling ball is thrown in a pattern its throw and catch heights are approximately the same or offset equally from the centre of the circles described by the hands such that the effects cancel themselves out (Figure 4). These conditions mean that the angle of trajectory of the throw of a juggling ball can be calculated simply from the ball's airtime (in seconds) and the horizontal distance it travels from hand to hand (throw width). It has been shown in previously that the airtime can easily be determined from the speed of the pattern or the throw height, and the weight and hold time of the throw. The throw width can be measured. The maximum vertical velocity of a ball (when it leaves the hand and again when it is caught) can be calculated from just the maximum height it achieves, which can be either set or determined. Using the vertical velocity and the angle of trajectory the actual maximum object velocity can be calculated.

Different combinations of throw rate, weight, height, hold time, angle, velocity and throw width can be calculated and compared from different starting points. Formulas for calculations of throw angle and velocity are provided for reference in the following section.

Variables & Formulas

When I first began to explore the mechanics of juggling, I decided that to best understand it, I would work out the theory myself and then seek corroboration and further insight. I discovered that there was not a standard terminology, so I used what I felt most comfortable with, and as such some of my terminology may not be entirely consistent with what is already 'out there'.

The flight of an object when it is thrown is utterly predictable, as described mathematically by the laws of projectile motion. Figure 8 shows the parabolic path of a ball thrown. The definitions of the variables and juggling formulas used for the slide rule scales are described below.

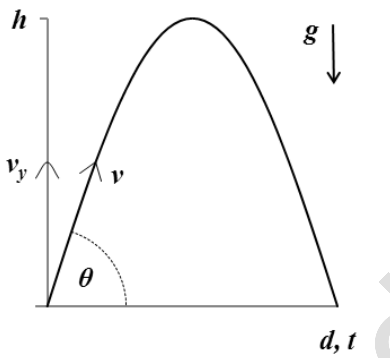


Figure 8: Parabolic flight of a thrown ball.

h	=	maximum height achieved (metres)
d	=	throw width (metres)
t	=	airtime (seconds)
θ	=	throw angle (degrees)
g	=	acceleration due to gravity (9.81 ms^{-1})
v	=	maximum velocity (ms^{-1})
v_y	=	maximum vertical velocity (ms^{-1})
n	=	throw weight
b_t	=	beat time (seconds)
f	=	throw rate (throws per second)
w_b	=	hold time (beats)

Beat time: $b = \dfrac{1}{f}$ Height: $h = \dfrac{t^2 g}{8}$ Vertical velocity: $v_y = \sqrt{2hg}$

Airtime: $t = b(n - w_b)$ Throw Angle: $\tan\theta = \dfrac{t^2 g}{2d}$ Velocity: $v = \dfrac{v_y}{\sin\theta}$

The Advanced Juggler's Slide Rule - Layout and Scales

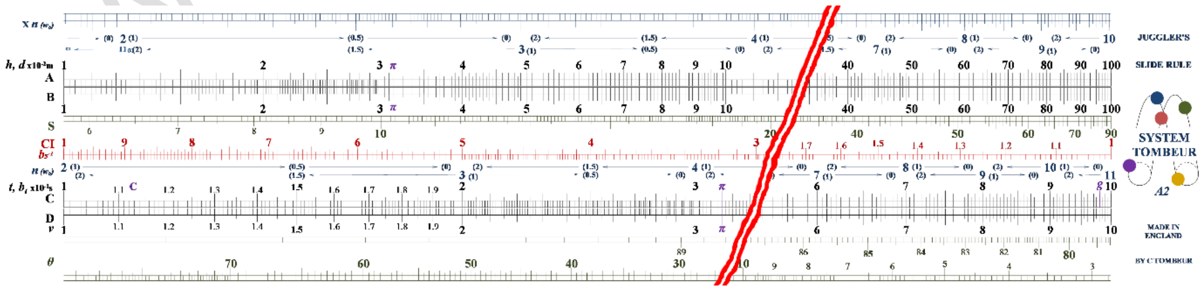


Figure 9: Advanced Juggler's Slide Rule scales (shown abridged)

The Advanced Juggler's Slide Rule allows calculations for hold times between zero and two beats so more complex regular patterns such as Arches (Figure 7) can be analysed. The single sided slide rule has 9 scales (Figure 9) - two on each of the top and bottom front faces of the stock, and five on the front of the slide. A cursor with single central hairline aids calculations and comparisons. Six of the scales are standard logarithmic slide rule scales (A, B, C, D, S & CI) and 3 are custom juggling scales (X, n(w_b) & θ). The standard scales are printed in black except for the red CI scale and green S scale. The *SYSTEM TOMBEUR* juggling scales are printed in blue except for the green θ scale (consistent with S, the other angle scale). The scales are described here from top to bottom.

Above the slide:

- '**X**' scale, blue, 2(1.097)-10(0.97) and 11(1.97)-11(1), also labelled '***n(w_b)***'. Logarithmic throw weight juggling scale for weights (n) 2 to 11. Weights 2 and 11 share the same graduation mark. Weights are marked at the base of the weight where hold time (w_b) is 1 beat. Graduations for hold times from (2) to (0) beats are marked between the weights. Note that the weight/hold ranges overlap and share the same graduations, and hold times (2) to (0) run in reverse direction to the weights (2 to 11).
- '**A**' scale, black, 1-100, also labelled '***h, d x 10^{-2}m***'. Standard slide rule logarithmic A scale, also used to represent throw height (h) in centimetres or metres, and throw width (d) in centimetres.

Slide:

- '**B**' scale, black, 1-100. Standard slide rule logarithmic B scale.
- '**S**' scale, green, 5°44'-90°. Standard slide rule logarithmic Sine scale, referenced against the C scale.
- '**CI**' scale, red, 10-1, also labelled '***bs^{-1}***'. Standard slide rule logarithmic reciprocal scale, also used to represent throw rate in beats per second.
- '**n(w_b)**' scale, blue, 2**(1)**-11**(1)**. Logarithmic throw weight juggling scale for weights (n) 2 to 11, like the 'X' scale. The scale uses the tick marks/graduations of the lower 'C' scale. Weights are marked at the base of the weight where hold time (w_b) is 1 beat. Graduations for hold times from (2) to (0) beats are marked between the weights. Note that the weight (hold) ranges overlap and share the same graduations, and hold times (2) to (0) run in reverse direction to the weights (2 to 10).
- '**C**' scale, black, 1-10, also labelled '***t, b_t x10^{-1}s***'. Standard slide rule logarithmic C scale, also used to represent airtime (t) and beat time (b_t) in seconds x10^{-1}.

Below the slide:

- '**D**' scale, black, 1-10, also labelled '***v***'. Standard slide rule logarithmic D scale, also used to represent maximum velocity (v).
- 'θ' scale, green, 2 ranges, upper 89°53'-78°30', lower 78°30'-2°50'. Logarithmic juggling scale to represent throw angle to the horizontal (θ).

In general, the scales only have value labels at the major tick mark divisions. In most cases the major divisions are then graduated with tick marks to two further levels, allowing 3 significant figures to be accurately represented in calculations. This level of accuracy can be insightful and highlight how small changes can have a large impact on a pattern, demonstrating what a precise art juggling can be when performed well.

As usual when performing calculations with slide rules, the left and right-hand end value marks of the conventional scales are used as indexes, decimal points and trailing zeros are ignored, and the correct magnitude is determined once the result has been obtained. If desired values are out of range against the setting of the slide, the cursor hairline is used to mark the index end of the slide within range, and the other index end is then aligned to the hairline and values read, which may need to be factored as appropriate.

The scales feature gauge marks to assist in calculations:

- **π** (3142) on A, B, C & D scales.
- **C** (1128) on C scale.
- **C_1** (3568) on C scale.
- **v_y** (4429) on C scale.
- **g** (981) on C scale.

All the calculations for juggling basics described above using the Simple Juggler's Slide Rule can also be performed on the Advanced Juggler's Slide Rules using the similar scales and methods. The following two sections describe how to perform juggling calculations and comparisons using the Advanced *SYSTEM TOMBEUR* slide rule for 'Pattern Speed, Height & Throw Weight' and 'Throw Angle & Velocity'. In each section the operations required to determine and compare values are described, with examples given throughout. There is some duplication of instructions with the usage the Simple rule above, which I feel is necessary given the different look of the Advanced rule with more, and differently labelled, scales.

Pattern Speed, Height & Throw Weight Calculations (Advanced model)

Pattern speed, height and throw weight are fundamentally interconnected. To understand the relationship of the variables and their use in calculations, this section has been further divided into topics. However, the reader is encouraged to study and refer to these topics within the context of this section as a whole.

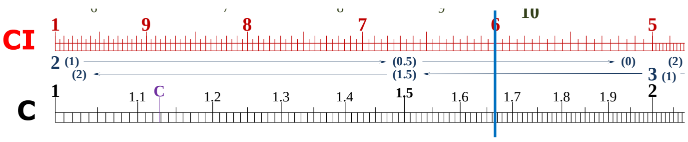
Pattern Speed

Pattern speed can either be represented on the C scale or the CI scale. The C scale represents the beat time in seconds. The CI scale, being the reciprocal of the C scale, represents beats (and therefore throws) per second.

Usually, the speed of a pattern is such that the beat time is between 0.1 and 1 second and this is represented on the C scale by using the values 1 to 10. Beats or throws per second can be directly read on the CI scale against beat time values on the C scale, or vice versa. The cursor hairline can be used to determine equivalents with greater accuracy if required.

Example 6: What is the beat time equivalent to throwing 6 balls per second?

*First ensure that all the scales are aligned under one another at the left-hand index end. Set the cursor hairline to the "6" tick mark on the CI scale, representing 6 throws per second. Read off the corresponding beat time of **0.167 seconds** under the hairline on the C scale.*



The C and CI scales can also represent beat times of 1 to 10 seconds (1 to 0.1 throws per second) or higher, but the resulting calculations must be factored accordingly.

Pattern Speed & Height Calculations

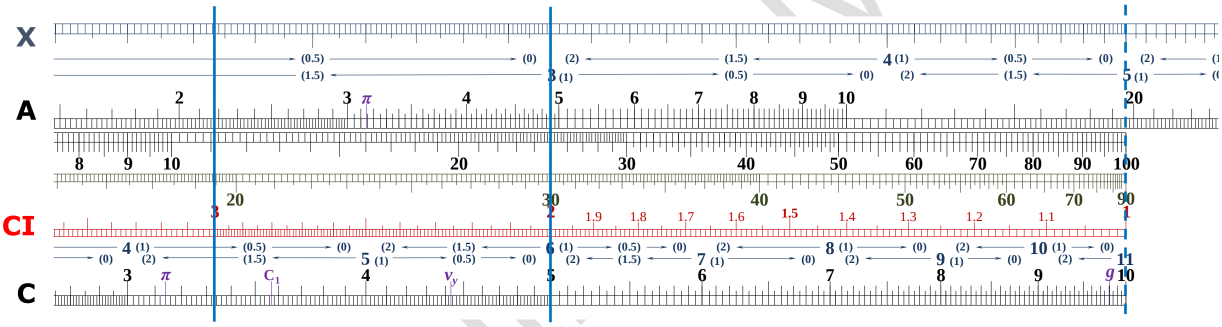
Calculations involving pattern speed, height, throw weights and hold times primarily use the X (weight/hold), A (height), CI (throw rate), and C (beat time) scales.

The X (throw weight) scale is labelled with throw weights (2 to 11) at the base of the weight, that is where the hold time is one beat. For example, weight 3 hold 1 beat is labelled as 3(1). Hold times from 0 beats to 2 beats can be found by using the graduations on the scale between the labelled weights. Arrows from each weight base to the left and right indicate increasing and decreasing hold times to their maximum of 2 beats and minimum of 0 beats for the weight. Care should be taken since whereas the weights increase from left to right, the hold times increase from right to left. Also, the weight hold ranges overlap, for example $3_{(0)}$ = $4_{(1)}$ = $5_{(2)}$ or $3_{(0.5)}$ = $4_{(1.5)}$.

To determine the maximum height achieved by a ball thrown at a certain weight and hold time, and at a particular beat time or throw rate in seconds, first find the weight and hold time value on the X scale and set the cursor hairline to it. Next set either the left or right-hand index of the slide to the cursor hairline. Find the pattern speed value on either the C or CI scale and set the cursor hairline to it. If the pattern speed value is such that its position on the slide is physically outside of the scale ranges on the stock, use the other index end of the slide. Read off the height on the A scale underneath the cursor hairline. Magnitudes for height are determined in the same way as with the Simple rule, as described above.

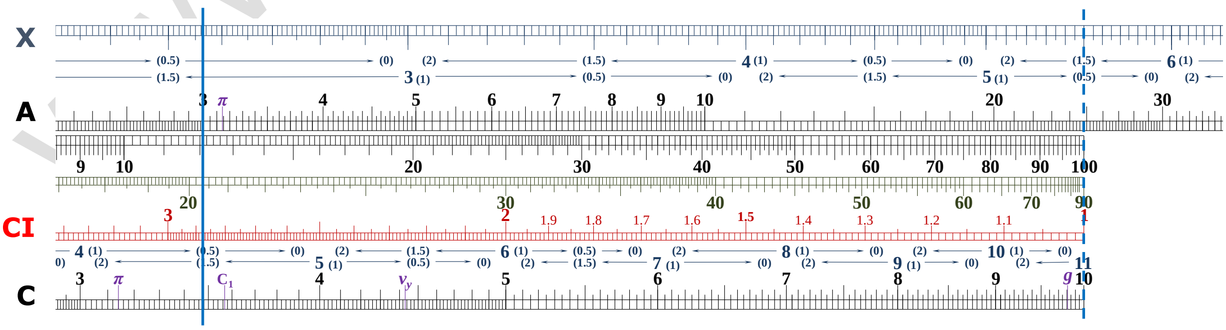
Once an index end of the slide is set to a weight/hold time value on the throw weight (X) scale, corresponding heights can be read for all pattern speed values on the C or CI scales that are in the A scale range. Similarly, pattern speeds can be read on the C or CI scales for specific heights on the A scale and setting the correct index end on the slide to the weight/hold time mark on the X scale will ensure that values can always be read.

Example 7: To what respective heights would the balls in a theoretically neat 5-ball cascade (with hold times of 1 beat) have to be thrown to achieve a rhythm of 3 balls per second, and then a throwing beat of 0.5 seconds?



*Set the cursor hairline over the "$5_{(1)}$" weight tick mark on the X scale representing a weight 5 throw with a hold time of 1 beat. Now set the right-hand index end of the slide under the cursor hairline. Next position the cursor hairline over the "3" tick mark on the CI scale on the slide to indicate 3 throws per second. Using the hairline the corresponding throw height of **2.18 metres** can now be read off on the A scale on the bottom of the upper stock face. Simply repositioning the cursor hairline over the "5" tick mark on the C scale gives the alternative throw height of **4.91 metres** on the A scale for a beat time of 0.5 seconds.*

Example 8: What throwing beat or rhythm is needed to sustain a 6-ball fountain when the balls are juggled/thrown to a height of 3 metres using a comfortable 1.5 beat hold time?



*Set the cursor hairline over the "$6_{(1.5)}$" weight tick mark on the X scale representing a weight 6 throw with a hold time of 1.5 beats. Set the right-hand index end of the slide under the cursor hairline. This time place the cursor hairline over the "3" tick mark on the A scale to indicate 3 metres. The corresponding throwing beat or rhythm of **0.348 seconds** is under the hairline on C scale, or **2.88 throws per second** on the CI scale.*

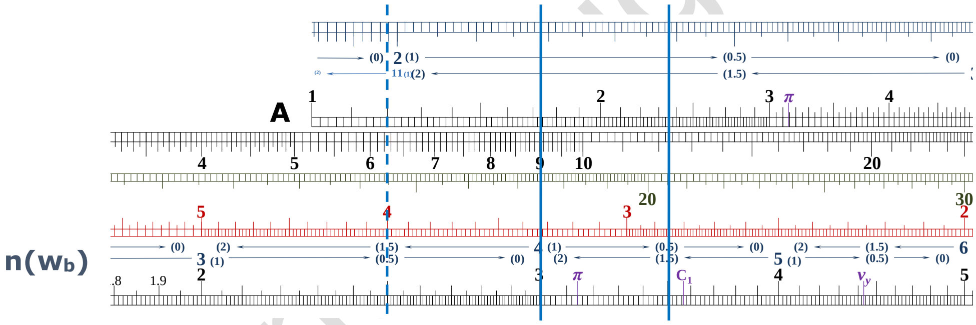
Pattern Height & Throw Weight/Hold Time Comparisons

The weight/hold time scale on the slide, $n(w_b)$, is the same scale as the weight/hold time scale (X) at the top of the upper face of the stock. However, the weights have been positioned to make use of the graduations on the C scale, and the values run from 2(1) to 11(1). Care should be taken when using the $n(w_b)$ scale on the slide, that the $n(w_b)$ scale value labels and corresponding C scale graduations are used correctly, and the inherent C scale values are ignored.

When a height has either been set or determined for a particular weight and hold time, it is very easy to quickly find all heights for all hold times (0 to 2 beats) for that weight/hold time and pattern speed by using the $n(w_b)$ scale on the slide.

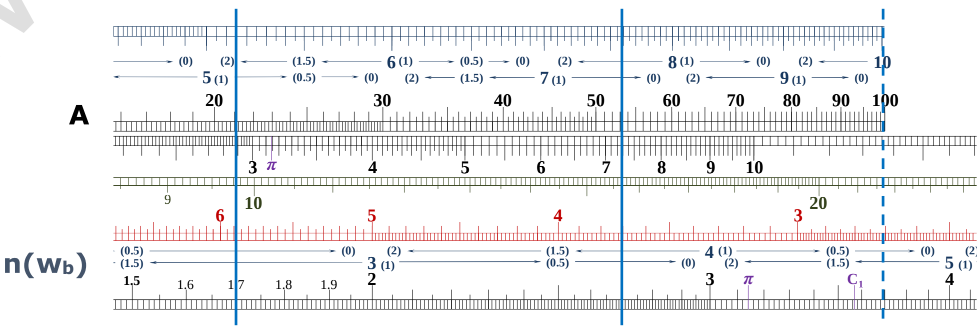
Position the cursor hairline over the starting height on the A scale. Move the slide so that the weight/hold marker on the $n(w_b)$ scale on the slide for the initial weight/hold time is underneath the cursor hairline. Heights can then be read on the A scale against all values of hold time on the $n(w_b)$ scale for the chosen weight, using the cursor to read across the scales. Further, for any height value on the A scale, any weight/hold time on the $n(w_b)$ scale can be set to it and heights read on the A scale against all weight/hold times on the $n(w_b)$ scale.

Example 9: A juggler comfortably juggles a 4-ball fountain with a hold time of 1.5 beats to a height of 1.2 metres. To demonstrate the pattern at the same speed with lower hold times of 1 beat and 0.5 beats, what respective heights would the balls have to be thrown to?



*Set the cursor hairline over the "1.2" tick mark on the A scale representing the height in metres. Move the slide so that the "4(1.5)" weight tick mark on the $n(w_b)$ scale is underneath the hairline, setting the weight of 4 with the initial 1.5 beat hold time. Now position the hairline over the "4(1)" weight tick mark on the $n(w_b)$ scale and read off the corresponding height of **1.73 metres** underneath the hairline on the A scale for a 1 beat hold time. Next move the cursor hairline over the "4(0.5)" weight tick mark on the $n(w_b)$ scale for a 0.5 beat hold time, and read off the corresponding height underneath the hairline on the A scale of **2.35 metres**.*

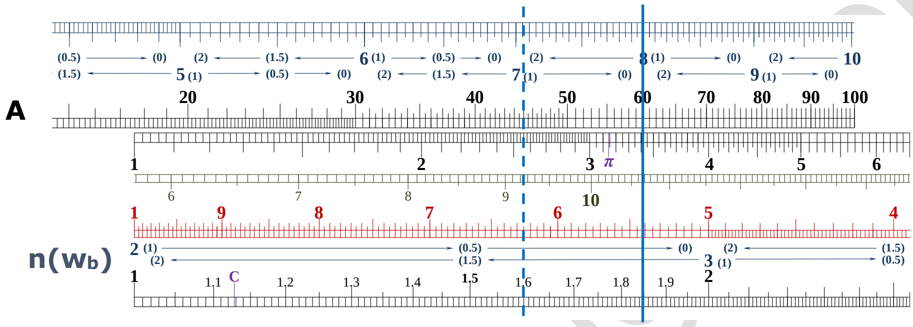
Example 10: In order to juggle the pattern 534 to a maximum height of 1 metre, if your comfortable juggling is at a hold time of 1.3 beats, what must the heights of your 3 and 4 throws be respectively to lock the pattern together?



Position the cursor hairline over the "100" tick mark on the A scale representing the height of 1 metre. Move the slide so the "5(1.3)" weight tick mark on the $n(w_b)$ scale is underneath the hairline, setting the weight 5 *throw to the height. Reposition the cursor hairline over the "3(1.3)" weight tick mark on the $n(w_b)$ scale and read **21.1 centimetres** on the A scale underneath the hairline for the height of the weight 3 throw. Now move the cursor hairline over the "4(1.3)" weight tick mark on the $n(w_b)$ scale. Read off the corresponding height of the weight 4 throw of **53.3 centimetres** on the A scale underneath the hairline.*

Using a similar technique, adjusted hold times can be determined for weights thrown to precise heights in a pattern. Set a weight/hold time on the $n(w_b)$ scale on the slide set to a height on the A scale, thus setting the pattern speed. The cursor can then be used to determine the hold time on the $n(w_b)$ scale required for a weight to be thrown to a specific height represented on the A scale.

Example 11: A 3-ball cascade is juggled with a 1.4 hold time to a height of 45 centimetres. If a ball was to be thrown out of the pattern as an 'over the top' throw to 60 centimetres, what would its hold time have to be reduced to?



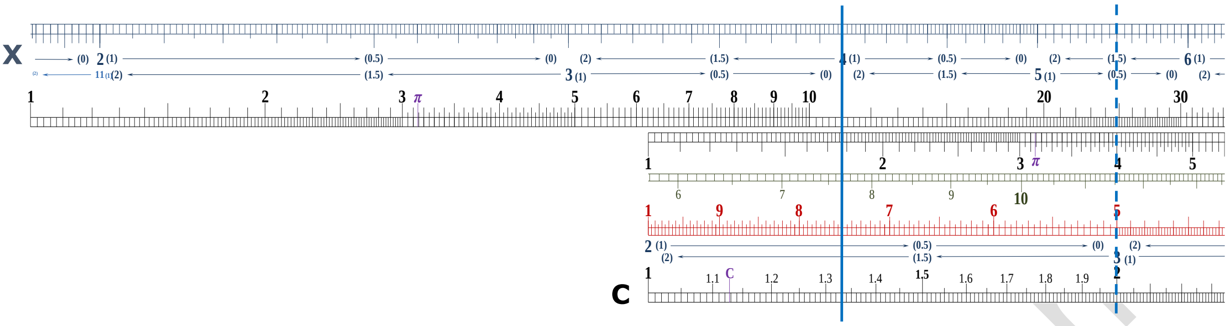
*Set the cursor hairline over the "45" tick mark on the A scale to represent the height of 45 centimetres. Next align the "3(1.4)" weight tick mark on the $n(w_b)$ scale underneath the hairline to set the weight. Move the cursor hairline over the "60" tick mark on the A scale to represent the new height of 60 centimetres. Read off a hold time of **1.15 beats** for the weight 3 over the top throw on the $n(w_b)$ scale under the hairline.*

<u>Patten Speed & Weight/Hold Time Comparisons</u>

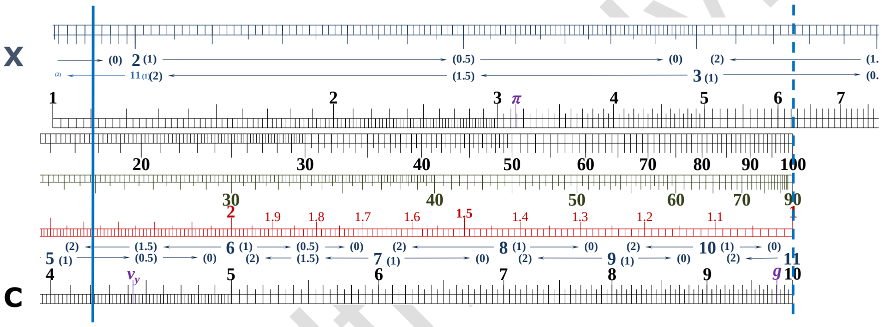
When a pattern speed has either been set or determined for a particular weight and hold time, it is very easy to quickly find all the pattern speeds for all the hold times (0 to 2 beats) for that weight and height by using the X scale (weight/hold time). To do this the C or CI scales are used for pattern speed, however due nature of the calculation their functions are reversed. The C scale is used to represent the pattern speed in throws per second, and the CI scale is used to represent the beat time in seconds. Care should be taken that the correct scale is used. It is useful to remember that longer hold times produce slower patterns and shorter hold times produce faster patterns to the same height.

Position the cursor hairline over the initial weight/hold time value on the X scale and move the slide so that the initial pattern speed on either the CI or C is underneath the cursor hairline. Pattern speeds can then be read on either the CI scale (beat time) or C scale (throws per second) against all values of the hold time on the X scale for the chosen weight, using the cursor to read across the scales. In fact, any pattern speed can be set to any weight/hold time on the X scale and the pattern speeds read off the C or CI scales against all the weight/hold times on the X scale.

Example 12: The height of a 5-ball cascade with a hold time of 0.5 beats and thrown at 2 beats per second is 6.21 metres. What would the rhythm need to be to maintain this height with a 4-ball fountain at 1 beat hold time, and an 11-ball cascade with 1.5 beat hold time?



*Set the cursor hairline over the "5(0.5)" tick mark on the X scale, representing the reference weight and hold time. Align the pattern speed of 2 throws per second to the weight/hold time by moving the slide so the "2" tick mark on the C scale is underneath the hairline. To find the pattern speed of the 4-ball fountain with hold times of 1 beat, reposition the cursor hairline over the "4(1)" tick mark on the X scale. Read off the throw rate of **1.33 balls per second** on the C scale under the hairline.*



*The cursor should then be repositioned over the weight/hold time mark on the X scale for the 11-ball cascade in order to read its pattern speed. However the weight 11 tick marks are not within the range of the scales on the slide, so the slide must be re-positioned first. Set the cursor hairline over the left-hand index end of the slide and then move the slide so the right-hand index end is underneath the hairline. Now position the cursor hairline over the "11(1.5)" tick mark on the X scale representing the weight 11 throws with 1.5 beat hold time. Underneath the hairline on the C scale, read off the pattern speed of **4.22 throws per second**.*

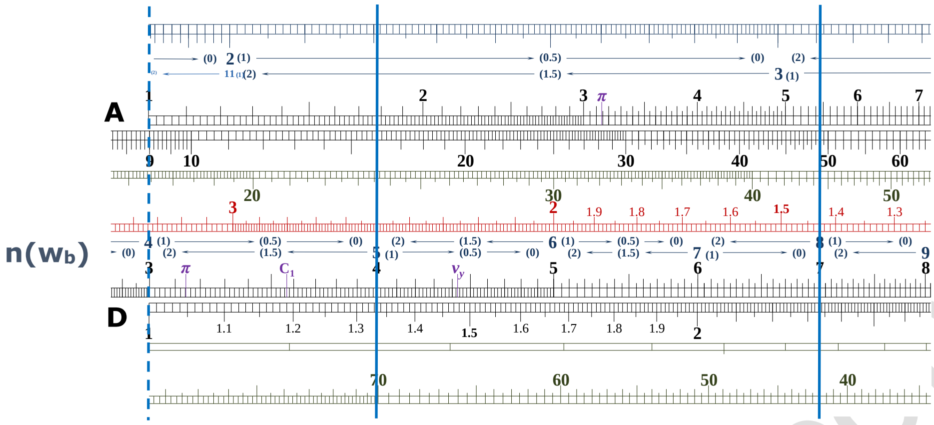
<u>Relative Comparisons</u>

Multiples of beat time or throw height for different throw weights and hold times can easily be determined using the $n(w_b)$, A and D scales, for weight/hold, height multiples and speed multiples respectively.

Position the slide so the left-hand index of the D scale below the slide on the front face of the stock is aligned to the lower weight/hold time mark on the $n(w_b)$ scale on the slide. Set the cursor hairline to any higher weight/hold time mark on the $n(w_b)$ scale and read off the corresponding value on the D scale. This value is the beat time multiple when these two different weight/hold times are thrown to the same height, that is, how many times faster the second pattern would be compared to the first. The value on the A scale at the cursor hairline is the height multiple for these two weight/hold times if they are thrown at the same pattern speed, that is, how many times higher the second weight/hold time throw would be compared to the first.

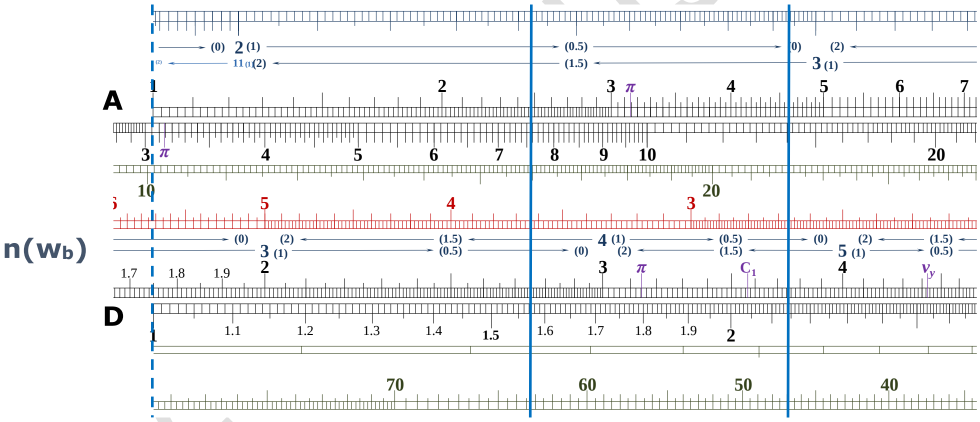
The same results can be determined by indexing the left-hand end of the slide to the X scale for the weight/hold times, and using the B and C scales for height multiples and speed multiples respectively.

Example 13: How many times faster than a 4-ball fountain would a 5-ball cascade and 8-ball fountain need to be juggled if their heights were all kept the same and they were all juggled at their base weight?



*Set the slide so the “4(1)” weight tick mark on the $n(w_b)$ scale, representing the weight 4 throws with 1 beat hold time, is aligned with the left-hand index end of the D scale. Position the cursor hairline over the “5(1)” weight tick mark on the $n(w_b)$ scale and read **1.33 times** under the hairline on the D scale for the weight 5, hold time 1 beat throws. Now move the cursor hairline over the “8(1)” weight tick mark on the $n(w_b)$ scale and read **2.33 times** on the D scale under the hairline for the weight 8 throws with a hold time of 1 beat.*

Example 14: In the pattern ‘534’ with a hold time of 1.25 beats, how many times higher than the ‘3’ would the ‘4’ and ‘5’ have to be thrown respectively?



*This time position the slide so the left-hand index of the D scale is aligned to the “3(1.25)” weight tick mark on the $n(w_b)$ scale, representing the weight 3 throws with hold times of 1.25 beats. All the weights have the same hold time, so now set the cursor hairline over the “4(1.25)” weight tick mark on the $n(w_b)$ scale and under the hairline on the A scale read **2.47 times higher** for the weight 4 throws. Reposition the cursor hairline over the “5(1.25)” weight tick mark on the $n(w_b)$ scale and read **4.59 times higher** for the weight 5 throws on the A scale under the hairline.*

Throw Angle & Object Velocity Calculations (Advanced model)

Calculation of throw angle and object velocity are made simple using the special scales and gauge marks. The methods used are described here by their respective topics.

<u>Throw Angle</u>

The angle of throw of a ball to the horizontal is calculated from its airtime and the horizontal distance it travels between being thrown and caught. This is accomplished using the C (airtime), A (distance) and θ (angle) scales.

First the airtime of the ball must be determined. This is simply its throw weight minus its hold time (both in beats), and either multiplied by the beat time or divided by the throw rate (in seconds or throws per second respectively). This calculation can be performed either mentally or by using the C or CI and D scales in conventional slide rule multiplication or division.

Once the airtime has been determined, set the cursor hairline to the horizontal distance in either centimetres or metres on the A scale. Position the slide so that the value for the airtime on the C scale is aligned to the cursor hairline. Move the cursor so the hairline is aligned with either the left or right-hand index end of the slide, whichever is within the bounds of the θ scale. The throw angle can then be read at the cursor hairline on either the upper or lower θ scale ranges. The rules to determine which of the θ scale ranges to use are as follows:

Airtimes between 0 and 1 second

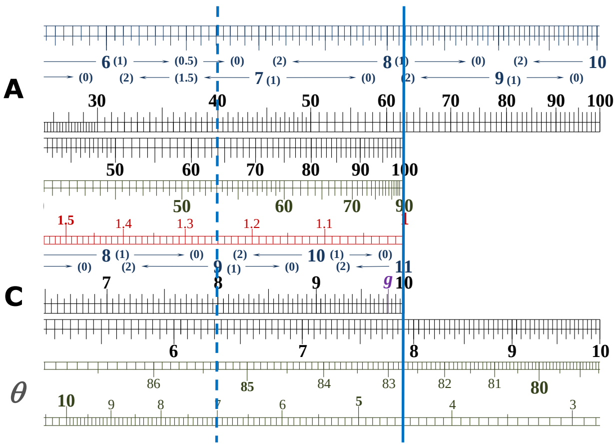
- For distance in *centimetres*, and when the slide protrudes from the stock to the *left*, then use the *upper* angle range.
- For distance in *centimetres*, and when the slide protrudes from the stock to the *right*, then use the *lower* angle range.
- For distance in *metres*, and when the slide protrudes from the stock to the *left*, then use the *lower* angle range.
- For distance in *metres*, and when the slide protrudes from the stock to the *right*, then the throw angle is outside of the scale ranges.

Airtimes between 1 and 10 seconds

- For distance in *centimetres*, and when the slide protrudes from the stock to the *left*, then the throw angle is outside of the scale ranges.
- For distance in *centimetres*, and when the slide protrudes from the stock to the *right*, then use the *upper* angle range.
- For distance in *metres*, and when the slide protrudes from the stock to the *left*, then use the *upper* angle range.
- For distance in *metres*, and when the slide protrudes from the stock to the *right*, then use the *lower* angle range.

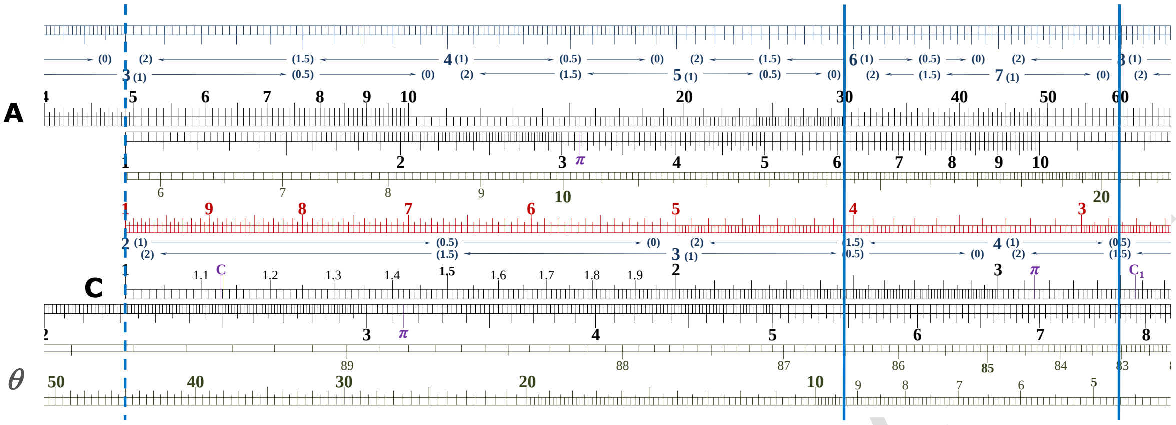
Similarly, using the same scales airtimes can be determined for known throw angles and displacements, or displacements calculated from throw angles and airtimes.

Example 15: What is the throw angle of a base weight 5-ball cascade juggled at 5 balls per second with the hands 40 centimetres apart?



First determine the airtime in seconds. The throw weight is 5 beats and base weight means one beat of hold time per throw, so the airtime is 4 beats. At a rate of 5 balls per second, the beat time is 0.2 seconds. Therefore the airtime of each ball is 0.8 seconds. Set the cursor hairline over the “40” tick mark on the A scale to represent the horizontal distance of 40 centimetres travelled by the ball. Position the slide so that the “8” tick mark on the C scale, representing the airtime of 0.8 seconds, is underneath the hairline. Now move the cursor so the hairline is over the right-hand index end of the slide. Read off the angle to the horizontal off the θ scale under the hairline. Since the horizontal distance is in centimetres, the *airtime is below 1 second and the slide is protruding to the left, then according to the rules the upper throw angle range must be used. Read **82°44'** underneath the hairline on the upper angle range.*

Example 16: What are the airtimes for throws made at a 55° angle if they were caught 30 and 60 centimetres away respectively?



*Set the cursor hairline over the "55" tick mark on the θ scale representing the throw angle. Align the left-hand index end of the C scale underneath the hairline. Reposition the cursor hairline over the "30" tick mark on the A scale and read off the corresponding airtime of **0.247 seconds** underneath the hairline on the C scale for a displacement of 30 centimetres. Move the cursor hairline over the "60" tick mark on the A scale and read **0.35 seconds** airtime underneath the hairline on the C scale corresponding to a 60 centimetre displacement.*

<u>Object Velocity</u>

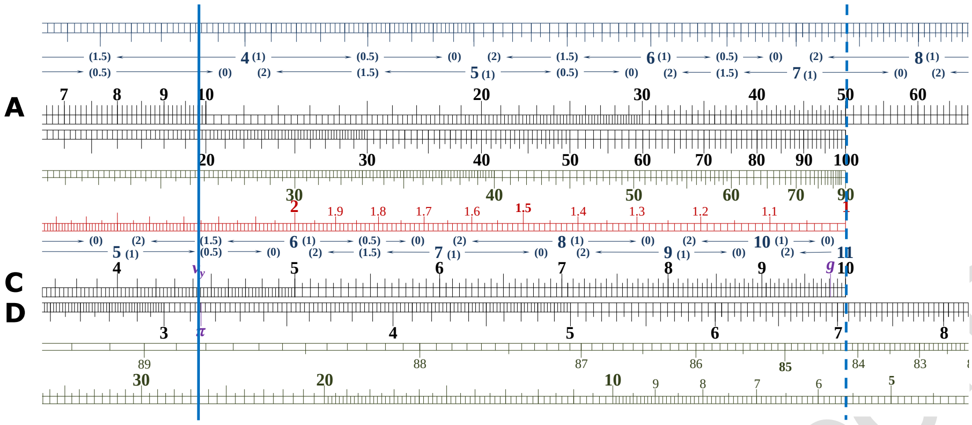
The maximum vertical velocity of a ball thrown to a certain height is calculated from the height it achieves and acceleration due to gravity (*g*), using the A (height) and D (velocity) scales and the v_y gauge mark on the C scale.

Set the left or right-hand index end of the slide to the throw height on the A scale. Use the index end that ensures that the v_y gauge mark on the C scale lies within the range of the D scale. At the v_y gauge mark on the C scale, read off the corresponding velocity on the D scale. The magnitude of the velocity is determined using the following rules:

- For height in *centimetres*, and when the slide protrudes from the stock to the *left*, velocity is in *metres per second (ms^{-1})*.
- For height in *centimetres*, and when the slide protrudes from the stock to the *right*, velocity is *10^{-1} x metres per second (ms^{-1})*.
- For height in *metres*, and when the slide protrudes from the stock to the *left*, velocity is in *10 x metres per second (ms^{-1})*.
- For height in *metres*, and when the slide protrudes from the stock to the *right*, velocity is in *metres per second (ms^{-1})*.

The calculation is easily reversed to find the maximum height achieved by a throw with a known initial vertical velocity. Align the v_y mark on the C scale to the velocity on the D scale and read off the height on the A scale at the appropriate index end of the C scale.

Example 17: If a ball is thrown to a height of 50 centimetres, what is the vertical component of its velocity when it caught?

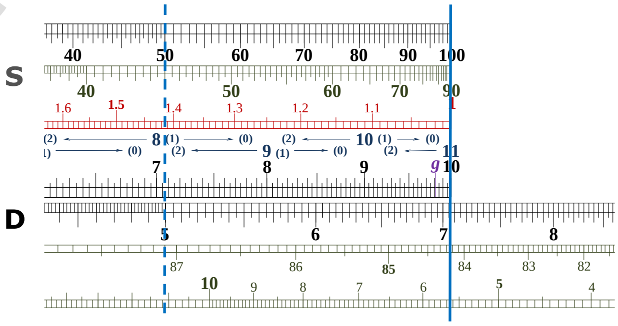


*Align the right-hand index end of the slide to the "50" tick mark on the A scale, representing a height of 50 centimetres. Find the v_y gauge mark on the C scale, and at this gauge mark read off the vertical velocity of **3.13 metres per second** on the adjacent D scale.*

The actual maximum velocity of a ball thrown is calculated from the angle of the throw to the horizontal and its maximum vertical velocity, using the S (angle) and D (velocity) scales. Set the cursor hairline to the vertical velocity on the D scale. Position the slide so the throw angle on the S scale is aligned with the cursor hairline. Read off the actual maximum velocity on the D scale at either the left or right-hand index end of slide, whichever is within the D scale range. The magnitude of the actual velocity is easily determined by common sense - it is always at least that of the vertical velocity, and not more than 10 times it if the throw angle is over 6°.

Example 18: If a ball thrown at 45° to the horizontal is caught having reached a vertical velocity of 5 metres per second, what was its actual velocity when it was thrown?

*Set the cursor hairline over the "5" tick mark on the D scale, representing the vertical velocity. Position the slide so that the "45" tick mark on the S scale, representing the throw angle, is aligned underneath the hairline. Read off the actual velocity of the ball when launched of **7.07 metres per second** on the D scale at the right-hand index of the slide.*



The velocity calculations can easily be reworked to find any missing third variable from the other two known values.

Conclusion

Most juggling, like most slide rules, looks complicated to the casual observer, but is actually elegantly simple. However, most slide rules are fairly easy to master whereas most juggling unfortunately is not. Most observers get overwhelmed after 3 objects and cannot follow what is going on anyway, let alone be able to fully appreciate the skill and dedication involved to achieve the effect.

My devices can describe the fundamental mechanics of many, often quite complex looking patterns with the ease of using a slide rule, and I like that I have created a new use for an elegant physical device which helps describe an elegant physical activity.

There is much more to juggling, such as multiple throw/catch sites, multiplex throws and catches, synchronous throws, different objects, and the effects of spin, but I have no plans for a third incarnation of Juggler's Slide Rules. It was fun and highly informative researching, designing, and building these slide rules, where I have learnt a lot about juggling, but probably a lot more about slide rules and in areas I did not anticipate. That others have been interested is also rewarding, especially as originally, I only really did all this for myself.

Meanwhile, juggling continues to be mastered all the time by people of all ages using, as it should do, the same old tried and tested method of just throwing things about. A chance encounter with an old slide rule not only sparked my interest to become an enthusiast but caught my imagination with my interest in juggling to wed these two unlikely bedfellows and show, once again, the versatility of 400-year-old slide rule technology.

Acknowledgements & Bibliography

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