Precision Slide Rules
for Mechanical, Constructional
and Electrical Engineers

Basic-Trig No. 1/60
Rietz No. 1/87, 111/87, 111/87 A,
4/87
Engineers Log Log No. 1/92
Electro No. 1/98, 111/98, 4/98
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Description of the Slide Rule

The following instructions will explain calculations with the CASTELL Slide Rules

- Basic-Trig No. 1 60
- Rietz No. 1 87, 111 37, 111 87 A, 4 87
- Engineers Log Log No. 1 92
- Electro No. 1 98, 111 98, 4 98

Please bear in mind the model No. of your slide rule (see imprint on body below the slide) and observe the specialties of this model dealt with in the following chapters.

The Slide Rules consist of

Body, Slide, and Cursor.

System Rietz

System Electro

Fig. 1

Fig. 2
Scales of the Slide Rules System Rietz No. 1/87, 111/87, 111/87 A, 4/87

The front contains the following scales, read from upper to lower:

<table>
<thead>
<tr>
<th>Scale Type</th>
<th>Symbol</th>
<th>Formula</th>
<th>Range</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inch Scale</td>
<td>Inch</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cube Scale</td>
<td>K</td>
<td>$x^3$</td>
<td>from 1—10—100—1000</td>
<td>upper body</td>
</tr>
<tr>
<td>Quadratic Scale</td>
<td>A</td>
<td>$x^2$</td>
<td>from 1—10—100</td>
<td>slide</td>
</tr>
<tr>
<td>Quadratic Scale</td>
<td>B</td>
<td>$x^2$</td>
<td>from 1—10—100</td>
<td></td>
</tr>
<tr>
<td>Reciprocal Basic Scale*</td>
<td>Cl</td>
<td>$1/x$</td>
<td>from 10 (on scale as 1) —1</td>
<td></td>
</tr>
<tr>
<td>Basic Scale</td>
<td>C</td>
<td>$x$</td>
<td>from 1—10</td>
<td>lower body</td>
</tr>
<tr>
<td>Basic Scale</td>
<td>D</td>
<td>$x$</td>
<td>from 1—10</td>
<td></td>
</tr>
<tr>
<td>Logarithmic Scale</td>
<td>L</td>
<td>$\log x$</td>
<td>from 0—1</td>
<td></td>
</tr>
</tbody>
</table>

and the back of the slide:

<table>
<thead>
<tr>
<th>Scale Type</th>
<th>Symbol</th>
<th>Formula</th>
<th>Range</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine Scale</td>
<td>S</td>
<td>$\sin 0.1x$</td>
<td>from 5° 44'—90°</td>
<td>back of slide</td>
</tr>
<tr>
<td>Sine-tangent Scale</td>
<td>ST</td>
<td>$\arcsin 0.01x$</td>
<td>from 34°—50° 43'</td>
<td></td>
</tr>
<tr>
<td>Tangent Scale</td>
<td>T</td>
<td>$\tan 0.1x$</td>
<td>from 5° 43'—45°</td>
<td></td>
</tr>
</tbody>
</table>

Scales of the Slide Rules System Electro No. 1/98, 111/98, 4/98

The front contains the following scales:

<table>
<thead>
<tr>
<th>Scale Type</th>
<th>Symbol</th>
<th>Formula</th>
<th>Range</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inch Scale</td>
<td>Inch</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exponential Scale</td>
<td>LL₂</td>
<td>$e^{0.1x}$</td>
<td>from 1.1—3.2</td>
<td>upper body</td>
</tr>
<tr>
<td>Quadratic Scale</td>
<td>A</td>
<td>$x^2$</td>
<td>from 1—10—100</td>
<td>slide</td>
</tr>
<tr>
<td>Quadratic Scale</td>
<td>B</td>
<td>$x^2$</td>
<td>from 1—10—100</td>
<td></td>
</tr>
<tr>
<td>Reciprocal Basic Scale*</td>
<td>Cl</td>
<td>$1/x$</td>
<td>from 10 (on scale as 1) —1</td>
<td></td>
</tr>
<tr>
<td>Basic Scale</td>
<td>C</td>
<td>$x$</td>
<td>from 1—10</td>
<td>lower body</td>
</tr>
<tr>
<td>Basic Scale</td>
<td>D</td>
<td>$x$</td>
<td>from 1—10</td>
<td></td>
</tr>
<tr>
<td>Exponential Scale</td>
<td>LL₃</td>
<td>$e^x$</td>
<td>from 2.5—10³</td>
<td></td>
</tr>
<tr>
<td>Efficiency Scale</td>
<td>W</td>
<td></td>
<td></td>
<td>with 1.98 and 4.98 on body, below slide</td>
</tr>
<tr>
<td>Pressure-drop Scale</td>
<td>V</td>
<td>$x^3$</td>
<td>from 1—10—100—1000</td>
<td>with 111.98 on lower body, Lower vertical edge of body</td>
</tr>
<tr>
<td>Cube Scale</td>
<td>K</td>
<td>$x^3$</td>
<td>from 1—10—100—1000</td>
<td></td>
</tr>
</tbody>
</table>

and on the back of the slide:

<table>
<thead>
<tr>
<th>Scale Type</th>
<th>Symbol</th>
<th>Formula</th>
<th>Range</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine Scale</td>
<td>S</td>
<td>$\sin 0.01x$</td>
<td>from 34°—90°</td>
<td>back of slide</td>
</tr>
<tr>
<td>Logarithmic Scale</td>
<td>L</td>
<td>$\log x$</td>
<td>from 0—1 (inverted scale)</td>
<td></td>
</tr>
<tr>
<td>Tangent Scale</td>
<td>T</td>
<td>$\tan 0.1x$</td>
<td>from 5° 43'—45°</td>
<td></td>
</tr>
</tbody>
</table>

* Cl means basic scale C inverted, from 1-10, running in opposite direction, from right to left.
Scales of the Slide Rules System Engineers Log Log No. 1/92:
Same as Electro models, but without W and V Scales.

Scales of the Slide Rule System Basic-Trig No. 1/60
Same as the Rietz models, but without scales K and ST.

The System of Slide Rule Calculations
If two ordinary scales or rules are placed one along the other, as shown in the following Fig. 3, then by proceeding to the right one will obtain the result: \(3.5 + 4.5 = 8\) (i.e. an addition), or by proceeding to the left: \(8 - 4.5 = 3.5\) (i.e. a subtraction).

Now, if two scales of the slide rule are placed along the other in the same way, then the result obtained will be:

\[3.5 \times 4.5 = 15.75\] (i.e. a multiplication)

or \(15.75 \div 4.5 = 3.5\) (i.e. a division)

Conclusion:
If on the slide rule two distances are added, this will result in a multiplication, if one distance is subtracted from the other, this will result in a division.
Reading the Scales

The correct reading of the figures is the most important problem in slide rule calculation. This, however, is by no means as difficult as it first appears, but will be easily mastered after a short period of practice. We must first of all study the system of graduation to become thoroughly conversant with the values of the individual small division marks in each section of the slide rule.

Please bear in mind:

The slide rule does not show the actual decimal value of a figure. Thus the figure 6 may stand as well for 0.6, 6, 60, 600, 6000 etc.

It is, therefore, advisable when setting and reading to ignore the position of the decimal point.

We read or set just as follows:

for 3\(\cdot\)65 = 3–6–5 (three, six, five)
for 5\(\cdot\)60 = 5–6 (five, six)

The position of the decimal point will be found afterwards by a rough mental estimate of the size of the answer.

Scales of the Slide Rules with 10" (25 cm) graduated length No. 1/60, 1/87, 111/87, 111/87 A, 1/92, 1/98, 111/98

(Slide Rules with 20" graduated length 4/87 and 4/98 see page 8).

The various scale sections, three in all, are subdivided at irregular intervals because the intermediate spaces become narrower towards the right. Let us best examine the two lower scales C and D. We discern:

Extended graduation — Scale section from 1\(\cdot\)2 — Scale section from 2\(\cdot\)4 — Scale section from 4\(\cdot\)10 — Extended graduation.

On the left of the 1 and on the right of the 10 there are so-called extended graduations, starting at the left at 0\(\cdot\)89 and ending at the right at 11\(\cdot\)2. These additional graduations are coloured red. They permit to carry on calculations with limit values just a bit below 1 (begin of scale) or above 10 (end of scale).

Part of Scale Section 1\(\cdot\)2

From division mark 1 to division mark 1\(\cdot\)1

10 subdivisions with 10 intervals each
(= 1/100 or 0\(\cdot\)01 for each division mark)

Here an accurate reading can be taken of the values corresponding to 3 places (e.g. 1\(\cdot\)0\(\cdot\)1). By halving the distances between two division marks we obtain an accurate reading to 4 places (e.g. 1\(\cdot\)0\(\cdot\)1\(\cdot\)5), whereby the last figure is always a 5.
Part of Scale Section 2-4

From division mark 3 to division mark 4
10 subdivisions with 5 intervals each
(= 1/50 or 0.02 for each division mark)

Fig. 6

Here an accurate reading can be taken to 3 places (3-8-2). The last figure is always an even one (2, 4, 6, 8). By halving the distances, one obtains the odd figures 1, 3, 5, 7, 9 (3-8-3).

Part of Scale Section 4-10

From division mark 8 to division mark 10
10 subdivisions with 2 intervals each
(= 1/20 or 0.05 for each division mark)

Fig. 7

Here an accurate reading can be taken to 3 places if the last figure is a 5 (9-0-5). By halving the distances one obtains even 4 places. The last figure is also always a 5 (9-0-7-5).

Other intermediate values must be estimated. Example: To set to 518, first find the value 5-1-7-5, by halving the distance between 515 and 520, and then move the cursor line slightly to the right.

Fig. 8

The user is advised to select numbers of his own for setting and reading and to exercise until he can do so with a fair degree of confidence.

For this purpose, use should be made not only of the cursor but also of the index figures 1 at both ends of the scale.
Scales of the Slide Rules with 20" (50 cm) graduated length No. 4/87, 4/98

The various scale sections, three in all, are subdivided at irregular intervals because the intermediate spaces become narrower towards the right. Let us best examine the two lower scales C and D. We discern:

Extended graduation — Scale Section from 1-2 — Scale Section from 2-5 — Scale Section from 5-10 — Extended graduation.

(The extended graduations allow to carry on calculations with limit values just a bit below 1 at the beginning and just a bit above 10 at the end of the scale.)

Scale Section from 1-2
This section is first divided into ten subsections which are marked 1-1, 1-2, 1-3, 1-4... to 1-9. Each of these subsections (i.e. tenths or 0.1) are in their turn divided into ten subdivisions (i.e. hundredths or 0.01); these are, however, not numbered, for lack of space. These subdivisions are halved again by small marks (i.e. two-hundredths or 0.005). The readings are therefore: 1-0-0-5, 1-0-1-0, 1-0-1-5, 1-0-2-0, 1-0-2-5... 1-9-9-5 and 2-0-0-0.

Scale Section from 2-5
Here again the first subdivision is in tenths, but with the exceptions of marks 2, 2-5, 3, 3-5, 4 4-5, and 5, these are not numbered. We must determine the other tenths ourselves, i.e. the values 2-1, 2-2, 2-3... till 4-7, 4-8, 4-9.
These tenths in their turn are divided into further tenths (i.e. hundredths = 0.01) but the centres between them are no longer halved. We thus have, starting with 2 and without using the decimal point, the following values: 2-0-0, 2-0-1, 2-0-2, 2-0-3, 2-0-4, 2-0-5, 2-0-6 etc. to 4-9-7, 4-9-8, 4-9-9, 5-0-0.

Scale Section from 5-10
From 5 onward the tenths are marked, with fifths in between them (i.e. fiftieths = 0.02). The readings are therefore: 5-0-0, 5-0-2, 5-0-4, 5-0-6, 5-0-8, 5-1-0, 5-1-2 etc. till 9-9-6, 9-9-8, 1-0-0.

If a value is to be set ending with an odd figure, we must set to the exact centre between two graduation marks. This can be done with a high degree of accuracy. Other intermediate values must be estimated.

Example:
To find 1-1-2-6 (see also first example under passage "Scale Section from 1-2") first set on 1-1-2-5 and then move the cursor line used for the setting or the index-1 of scale C slightly to the right, whereby the value 1-1-2-7-5 will serve as a foothold as this lies in the middle between 1-1-2-5 and 1-1-3.
**Preliminary Notice:** For setting and reading of values we always use the long centre line of the cursor (hereafter called “cursor line”), and the index-1 at the beginning or the index-100 (or index-10) at the end of the main scales A, B, C, D.

**Multiplication**

Multiplication is carried out on the slide rule by adding together the distances corresponding to the numbers of the factors. We mainly use the basic scales C and D.

Example: 2.45 \( \times 3 = 7.35 \)

![Fig. 9](image)

Set 1 on beginning of slide (C 1) over 2.45 on the lower scale on body (D 245), place the cursor line over 3 on the lower scale of the slide (C 3) and read the product, 7.35, under the cursor line on the lower scale of body (D 735).

This multiplication can also be performed on scales A and B, however, with a slightly smaller degree of accuracy.

Example: 2.5 \( \times 3 = 7.5 \)

![Fig. 10](image)

Set 1 on beginning of slide (B 1) under 2.5 of the upper scale on body (A 25), place the cursor line over 3 on the upper scale of the slide (B 3) and read the product, 7.5, under the cursor line on the upper scale of the body (A 75).

When calculating with the main scales C and D, one finds that sometimes the second factor falls beyond the end of the scale and cannot be set.

Example: 7.5 \( \times 4.8 = 36 \)

![Fig. 11](image)

In this case move the slide to the left and place the right-hand index line C 10 over the first factor (D 7.5). Then move the cursor line over C 4.8 and read the result 36, on D. This process is called: “transposing the slide.”
Division

Division is carried out on the slide rule by subtracting the distance corresponding to the divisor from the distance corresponding to the dividend.

Example: $9.85 \div 2.5 = 3.94$

Set the cursor line over the dividend 9.85 on scale D and move the divisor 2.5 on scale C under the cursor line. Read the quotient 3.94 on scale D and C 1.

Here again it may occur that the slide has to be moved so far to the left that the result under C 1 cannot be read off; then the result can be found at the right scale end under C 10.

Example: $210 \div 28 = 7.5$

Set C 28 over D 210 with the aid of the cursor line. C 1 extends so far to the left that the result cannot be read off below it. Transpose the slide and find the result 7.5 underneath C 10.

Compilation of Tables

Example: Convert yards into metres.

We know the parity: 75 m are equal to 82 yards. Place C 75 over D 82. This automatically produces a table from which the following readings can be taken: 42 yds. are 38.4 m, 28 yds. are 2.56 m, 640 yds. are 585 m, 16 m are 17.5 yds., 128 m are 140 yds., etc.

The reading is taken with the aid of the cursor line. We set the cursor line over the known number of meters on C and find the corresponding number of yards also under the cursor line, on scale D, and vice versa.

If we do not know a certain parity (e.g. 75 English lbs. = 34 Kgs.), but only know that 1 lb. is equivalent to 0.454 Kgs., then we place C 1 or C 10 over D 4.5-4 and then also have the conversion of lbs. into Kgs., whereby the lbs. are found on C and the Kgs. on D.
Compound Multiplication and Division

\[
\begin{aligned}
13.8 \times 24.5 \times 3.75 &= 0.491 \\
17.6 \times 29.6 \times 4.96 &= 0.491
\end{aligned}
\]

Always start with a division, then follow with multiplications and divisions in alternate succession. The intermediate results need not be read off. First we place D 1-3-8 and C 1-7-6 one under the other (division). Do not read off the answer — about 0.8 under C 10 and D — but multiply it immediately by 24.5, by placing the cursor line on C 2-4-5. The result (about 1.9 on D) is simply divided by 29.6, by keeping the cursor line firmly in its position and sliding C 2-9-6 under it. Then comes the multiplication of the result (0.65 under C 10 on D) by 3-7-5 and finally the division by 4-96 in the same manner as described above. Only then do we read off the figures of the final answer, 4-91, under C 10 on D, and our rough calculation shows us that the actual answer is 0.491.

These calculations can likewise be carried out on the upper scales A and B.

Squares and Square Roots

The square of any number is obtained by setting the cursor line over this number on scale D, the result is shown above it on scale A, under the cursor line. We can, however, also use the first and last graduation marks at either end of the scales.

Example: \(3^2 = 9\)

\[
\begin{aligned}
\sqrt{3.65} &\quad & \sqrt{9} &\quad & \sqrt{36.5} \\
1.91 &\quad & 3 &\quad & 6.04
\end{aligned}
\]

When setting and taking readings one should get accustomed to work only with groups of figures and determine the decimal value thereafter by a rough estimate. The figures should be rounded off liberally, as this rough estimate is merely made to avoid getting tenths or tenths of the actual cursor.

Examples: \(1.345^2 = 1.81; 4.57^2 = 20.9; 0.765^2 = 0.585; 67.3^2 = 4530.\)
Square Roots are extracted by reversing this procedure. If, therefore, the cursor line is placed on a figure on A, then
the square root can be read off on D, underneath it. Extracting square roots is, however, not as simple as finding the
square, as may be seen from the following example: Extract \( \sqrt[4]{3.65} \). Place the cursor line on A 3-65 and find under-
neath on D the square root 1.91. As we have learned that the position of the decimal point is not shown on the slide
rule, we could also set the cursor line over A 365 (in the second half of scale A between 10 and 100). Underneath
we would find the root 6.04, which is obviously the wrong result.
Before extracting the square root, we must decide if the number must be set on the right or the left half of the
scale. At the left are the numbers from 1 to 10, and at the right those from 10 to 100.
Exercises: \( \sqrt[2]{4.56} = 2.14; \sqrt[6]{6.88} = 2.77; \sqrt[7]{45.3} = 6.73; \sqrt[10]{70.8} = 8.41. \)
If the number lies outside the scale range 1 to 100, it should be factorised by hundreds to bring the significant figu-
res within these limits.
Examples: \( \sqrt[3]{1936}. \) We factorise \( \sqrt[3]{1936} = \sqrt[3]{100 \times 19.36} = 10 \times \sqrt[3]{19.36} = 10 \times 4.4 = 44 \)
\( \sqrt[4]{0.543} = \sqrt[4]{54.3 \div 100} = \sqrt[4]{54.3 \div 10} = 7.37 \div 10 = 0.737; \sqrt[3]{0.00378} = \sqrt[3]{37.8 \div 10000} = \sqrt[3]{37.8 \div 100} = 6.15 \div 100 = 0.0615 \)
\( \sqrt[5]{145.8} = \sqrt[5]{100 \times 1.458} = 10 \times \sqrt[5]{1.458} = 10 \times 1.207 = 12.07; \sqrt[5]{507000} = \sqrt[5]{10000 \times 50.7} = 100 \times \sqrt[5]{50.7} = 100 \times 7.12 = 712 \)

The Reciprocal Scale CI

It corresponds to the subdivisions on scale C and D, but runs in the opposite direction, from right to left. The scale CI,
in conjunction with the scale C, provides a table of reciprocal values.
1. In order to find the reciprocal value \( \frac{1}{a} \) for any given number a, find a on C (or CI) and read above it on CI
(or below it on C) the reciprocal value. Reading off is done therefore without any movement of the slide and
entirely by setting the cursor line.
Examples: \( 1 \div 8 = 0.125; \ 1 \div 2 = 0.5; \ 1 \div 4 = 0.25; \ 1 \div 3 = 0.333. \)
2. To find \( \frac{1}{a^2} \) move the cursor to a on scale CI and read above it on B the result.
Example: \( \sqrt[5]{24} = 0.168. \) Estimated answer — less than \( \frac{1}{5} = 0.2. \)
3. To find \( \frac{1}{\sqrt{a}} \), set the cursor line to a on scale B and find below it on CI the result.
Example: \( \frac{1}{\sqrt[5]{27}} = 0.191. \) Estimated answer — less than \( \frac{1}{5} = 0.2. \)
4. Products of three factors can generally be reached with one setting of the slide. One sets, by means of the cur-
sor, the first two factors against each other on D and CI respectively, moves the cursor to the third factor on C
and reads below it on D, the final product.
Example: \( 0.66 \times 20.25 \times 2.38 = 31.8. \)
5. **Compound Multiplication and Division** can also be calculated conveniently with the scale C1.

Example: \( \frac{364}{32 \times 46} = 2.472 \)

One sets by means of the cursor the figures 3-6-4 on D against 3-2 on C. It is not necessary to read the intermediary result. Move the cursor line over 4-6 on scale C1, which is the same as multiplying by \( \frac{1}{4.6} \) (reciprocal value). The result 2.472 is then found under the cursor line on scale D.

### Cubes and Cube Roots

The cube scale consists of three equally long sections 1-10, 10-100, and 100-1000 and is used in conjunction with scale D. The cursor is set over the base number on D and the cube is read off on K.

On the slide rules 1.98, 11.98 and 4.98 the cube scale K is situated on the lower vertical edge of the rule and readings are taken against the lined tongue on the lower edge of the cursor.

Examples: \( 1.54^3 = 3.65; \ 2.34^3 = 12.8; \ 4.2^3 = 74.1. \)

When extracting a cube root, we use the opposite procedure. We set on K and read off on D.

Examples: \( \sqrt[3]{466} = 1.67; \ \sqrt[3]{29.5} = 2.09; \ \sqrt[3]{192} = 5.77. \)

When the base number is below 1 or above 1000, we must factorise it by powers of 10 to bring it within the interval of 1-1000 (similar to the method applied for square roots, see page 12).

If the cube scale K is employed with the Square Scale A, powers with the exponents \( \frac{2}{3} \) and \( \frac{3}{2} \) can be obtained with the aid of the cursor line.

Example: \( 7.5^\frac{3}{2} = 20.5 \) Set the cursor line over 7.5 on A and read underneath on K the answer 20.5.

Example: \( 132^\frac{1}{3} = 25.9 \) Set the cursor line over 132 on K and read underneath on K the answer 26.9.

13
The Logarithmic Scale L

On Slide Rules 1/87, 111/87, 111/87 A, 4/87

On these models the mantissa scale is situated at the lower face of the rule. The scale enables readings of the mantissa of the common logarithms. It cooperates with scale D.

Examples: \( \log 1.35 = 0.1303 \); \( \log 0.237 = 0.375 - 1 \).

Set the cursor line over the number on scale D and find the mantissa underneath, on scale L. If the mantissa is given, the desired number will be found by reversing this operation.

On Slide Rules 1/60, 1/92, 1/98, 111/98, 4/98

On these models the mantissa scale is found on the back of the slide. For calculations the entire slide rule is turned over and the slide must always be moved towards the right.

Example: Given logarithm 2.374. Find the number.

Turn the slide rule over and move the slide to the right until the mantissa 3-7-4 appears on scale L (lg) under the lower index line. Turn the slide rule against the front and find 2-3-6-6 under C 1. The characteristic was 2, therefore the desired number is 236.6.

The logarithm to any number is found by reversing this procedure. The characteristics or decimals are found as in usual logarithmic calculations (see Fig. 16).

Example: \( \log 1.35 = 0.1303 \).
The Trigonometrical Scales S, T and ST

The Sine Scale S on the Rietz Slide Rules 1/87, 111/87, 111/87 A and 4/87 cooperates with the C scale. The sine is read off on C over D 10 or D 1, but the values must be taken as tenths.

Example: \( \sin 32^\circ = 0.53 \)

Turn over the slide rule and pull the slide out towards the right until \( 32^\circ \) of the (back figured) sine scale appears under the righthand upper index in the slot. Then turn over the rule again and find the value 5-3 over D 10. Result: 0.53.

One could as well have pulled the slide out to the left and placed \( 32^\circ \) under the left hand upper index. The result is then found over D 1. This setting is preferably used in case of small angles.

Example: \( \sin 6^\circ 50' = 0.119 \).

Pull the slide to the right until \( 6^\circ 50' \) comes underneath the left upper index, turn over and find the figures 1-1-9, over D 1. The cosine of an angle is found by using the sine scale from right to left (red ciphers).

With slide rules 1/60, 1/92, 1/98, 111/98, 4/98 one proceeds in a similar manner. On these models the sine scale works in conjunction with the B scale. Mark A 1 or A 100 of the scale A show the results which have to be divided by 100. Scale B thus extends from 0.01—1.0.

* If you have bought a slide rule with 400° circle, please study the comments at the top of page 18.
The Tangent Scale $T$ of all slide rules dealt with in this booklet (excepting 4:98) works in conjunction with the lower scales $C$ and $D$ which is to be read from 0:1-1, but every number found must be divided by 10. When tangent-readings are taken, the setting can only be made at the left-hand lower index in the slot at the back of the rule. The reading is taken in a similar way as explained for the sine scale.

Example: $\tan 7^\circ 40' = 0.1346$.

Turn the rule and pull the slide towards the left until $7^\circ 40'$ appears below the left lower index line of the $T$ scale. Turn the rule again and read the value 1-3-4-6 over D 1 on C. Conversely it is also possible to set $\tan 0.1346$ over D 1 and to find on the reversed slide rule the corresponding angular value $7^\circ 40'$. The cotangent is found on D under C 1 or C 10 as 7.43.

Fig. 18

Tangents and cotangents of angles over $45^\circ$ are found by means of the equations:

$$\cot \alpha = \tan (90^\circ - \alpha) \text{ or } \cot \alpha = \frac{1}{\tan \alpha}$$

In case of the latter equation the tangent of a given angle — e.g. $25^\circ 40'$ — is placed over the index on the back of the slide, the tangent is read off over D 1 as 0.438, and the cotangent is found by means of the cursor line on the reciprocal scale C 1 as 2.28.

On the slide rule No. 4:98 the tangent scale $T$ works in conjunction with scale B. The setting is done as explained above, but readings are taken under A 1 on B, and these must be divided by 100.
The Sine-Tangent Scale ST for small angles from 34' to 5° 43' are provided on the Rietz models 1/87, 111/87, 111/87 A and 4/87. For such small angles the difference between sine and tangent is insignificant, we can therefore use the ST scale simultaneously for both functions. We use the right-hand lower index line, dividing the values founds on C by 100. The cotangent values read off on D must be multiplied by 10.

Example: sin 3° 38' or tan 3° 38' ≈ 0.0634.

Set the angle 3° 38' of the ST-scale over the right-hand lower index line, reverse the slide rule again and read off the result, 0.0634, over D 10 on C.

If many sine and tangent values are to be determined, one simply reverses and inserts the slide in such a manner that the sine scale S is opposite scale A and the tangent scale opposite the scale D. This forms a table, and all desired values can be set and read off by means of the cursor line.

Marks for trigonometrical calculations

The marks o' and o'' are provided on scales C and D and are used for reading the functions of very small angles. With such small angles, there is no difference, for practical purposes, between the trigonometrical functions sin and tan and the arc.

Mark o' is found between 34 and 35 on scales C and D and is used if the angle is given in minutes.

Example: sin 17' ≈ tan 17' ≈ arc 17' = 0.00495.

Set the mark o' over D 17 and read the function under C 10 on D.

Mark o'' is found between 2 and 2.1 on scales C and D and is used if the angle is given in seconds.

Example: sin 43'' ≈ tan 43'' ≈ arc 43'' = 0.0002085.

Set the mark o'' over D 43 and find the function under C 1 on scale D.
Mark θ on Slide Rules with New Angle Scales (400° circle)
The reading of the angular functions is done in just the same manner as on slide rules with a 360° circle, but there will be different values as results. Here the mark θ between C 63 and C 64 is used for small angles (given in centesimal minutes and seconds).
Examples: \( \sin 32^d = 0.482; \sin 63^d = 0.836; \tan 44^d = 0.827; \tan 12\cdot40^d = 0.1973; \)
\( \sin 0.17^d \approx \arctan 0.17^d \approx \tan 0.17^d = 0.00267. \)

Marks for Constant Values

Some frequently used constant values are specially marked:

\( \pi = 3.1416 \) on scales A, B, C1, C, D
\( M = \frac{1}{\pi} = 0.318 \) on scales A and B
Index line for \( \frac{\pi}{4} = 0.785 \) on A and B

Cross section marks C and C1 on scale C are used for calculation of areas of circles with a given diameter.
\( C = \sqrt{\frac{4}{\pi}} = 1.128 \) on scale C
\( C_1 = \sqrt{\frac{40}{\pi}} = 3.57 \) on scale C

Examples for the use of marks C and C1:
Place mark C over a given diameter, e.g. 2.82 on D, and read on A over B 1 (or B 10) the area 6.24 sq. inch.
With this area the contents of a cylinder can be found by multiplying it by the height (e.g. 4 inch):
We keep the slide in the same position (A 6.24 over B 1), move the cursor line over B 4 and read over it on A the contents of the cylinder (with \( d = 2.82 \) and \( h = 4 \)) = 25 cu. inch.
The mark C1 (do not mistake for beginning of slide C1) serves the same purpose as mark C. Mark C1 is used whenever for the setting of mark C the cursor would have to be moved too far to the right.
On slide rules “Electro” No. 1/98, 111/98, 4/98 we find on A and B the additional marks 746 and Cu (in black and red colouring). These will be explained later, on page 26.
The Multi-Line Cursor

In addition to the central main cursor line, cursors of the “Rietz” and “Electro” system slide rules have four further lines marked d, s, kW, HP (see Fig. 19) which make possible several very frequent and important mathematical operations:

1. **Calculation of the area of a circle from a given diameter**

   Example: Diameter = 4.8 inches.

   Set the right-hand lower cursor line (d₁) or main cursor line (d₂) over the diameter 4.8 inches on the lower scale D and read on the upper scale A, under the main cursor line (s₁) — or under the left-hand upper cursor line (s₂), if d₂ has been used for the setting — the required area 18.1 sq. inches.

2. **Calculation of the volume of a cylinder**

   Example: Diameter = 4.8 inches; height = 3.24 inches.

   First find the circular area, 18.1 sq. inches, as explained above, then multiply by the height 3.24 inches in the usual manner. The required volume = 58.64 cu. inches.

3. **Changing of Watts into HP and v.v.**

   Example: How many Watts are 48 HP?

   Set the cursor line “HP” over 48 on scale A. Under the cursor line “kW” will be found 35,800 Watts on scale A. For more accurate calculation set the cursor line “HP” over 48 on D. Under the cursor line “kW” will be found 35,808 Watts on scale D.

   Cursors for slide rule No. 1/92 “Engineers Log Log” are provided with three lines only — the central main cursor line and two further lines marked d and s, for the calculation of circular cross sections from given diameters, and for general calculations.

   Cursors for slide rule No. 1/60 “Basic Trig” have only one line for general calculations.
The Log-Log-Scales LL<sub>2</sub> and LL<sub>3</sub>

(on slide rules 1/92, 1/98, 111/98, 4/98)

The log-log scale begins in the left top corner with 1·1 and extends to 3·2 (LL<sub>2</sub>), and then continues below from the left, the portion from 2·5 to 3·2 being repeated, and ends on the right below with 100,000 (LL<sub>3</sub>). These two portions of the log-log scale are arranged in relation to each other and to the lower scale in a particular manner, which renders numerous applications possible.

![Log-Log Scales Diagram](image)

Under each number of the upper log-log scale stands on the lower scale its tenth power. The cursor line is used for setting.

Example: 1·1072<sup>10</sup> = 2·769 (Fig. 20a); 1·204<sup>10</sup> = 6·4 (Fig. 20b); 1·443<sup>10</sup> = 39·15 (Fig. 20c); 0·1443<sup>10</sup> = \( \frac{1·443}{10} \) = \( \frac{39·15}{10^{10}} \)

Over every number on the lower log-log scale (LL<sub>3</sub>) stands on the upper log-log scale (LL<sub>2</sub>) the tenth root.

Example: \( \sqrt[10]{3·4} = 1·1302 \) (Fig. 20d); \( \sqrt[10]{4·41} = 1·16 \) (Fig. 20e); \( \sqrt[10]{75} = 1·54 \) (Fig. 20f).
Powers of e

The powers of e (base of natural logarithm $e = 2.71828\ldots$) are obtained by setting the Basic Scale (D scale) to the exponent by means of the cursor. The power of e is then read from one of the LL scales; for the D scale, the range 1 to 10 is used when the reading is taken from the LL$_3$ scale and the range 0-1-1 when it is taken from the LL$_2$ scale.

Examples: $e^0 = 1$; $e^{1.61} = 5$; $e^{0.161} = 1.1748$.

$e^{6.22} = 500$; $e^{0.622} = 1.862$; $e^{0.336} = 1.4$.

$e^{12.5} = e^{10 + 2.5} = e^{10} \times e^{2.5} = 22000 \times 12.2 = 286500$.

Example: To calculate the clamping force in the "take-up band" (D$_{tub}$) of a band brake of which the band goes round the drum twice. (D$_{tub}$ = "take-off band").

$D_{tub} = 22$ kg; $\alpha = 2 \times 360^\circ = \pi = 12.56$; coefficient of friction $\mu = 0.18$; $\mu \times \alpha = 12.56 \times 0.18 = 2.261$;

$D_{tub} = D_{tub} \times e^{\mu \alpha} = 22 \times e^{2.261} = 22 \times 9.60 = 211.2$ kg.

If the exponent is negative, we determine $e^n$ and then calculate the reciprocal, e.g.:

$e^{-2} = \frac{1}{e^2} = 1.39 = 0.1353$; $e^{-6.22} = \frac{1}{e^{6.22}} = \frac{1}{500} = 0.002$.

Roots of e

Write the root as a power with a reciprocal exponent, and then proceed as above.

Examples: $\sqrt[4]{e} = e^{0.25} = 1.284$; $\sqrt[8]{e} = e^{0.125} = 54.6$.

$\sqrt[125]{e} = e^{0.06} = 1.333$; $\sqrt[30.1]{e} = e^{0.06} = 3000$.

$\sqrt{e} = e^{0.8} = 2.225$; $\sqrt{e} = e^{16.66} = e^{8.33} \times e^{8.33} = 4165 \times 4165 = 17,350,000$. 

Fig. 21
In this case, particularly where fractions are involved, it is of advantage to use the Reciprocal Scale Cl, e.g. $$\frac{1}{e} = 1.5853$$ (Fig. 21a).

**Exponential Equation** \( e^x = a \).

To solve the exponential equation \( e^x = a \), set the LL scale to \( a \) and find \( x \) on the lower scale graduation D.

Examples: \( e^x = 20.1; x = 3 \) (Fig. 21b).
\( e^x = 11; x = 2.4 \) (Fig. 21c).
\( e^{\frac{1}{y}} = 1.485; y = 2.529 \) (Fig. 22a).

![Fig. 22](image)

**The Natural Logarithms**

The natural logarithms are found when changing over from the LL scales to the Basic Scale. As regards the Basic Scale ranges (of numbers of places) the same applies, mutatis mutandis, as explained in the section on "Powers of e", e.g.

\[
\ln 94 = 4.54 \text{ (Fig. 22b).}
\]
\[
\ln 1.87 = 0.626 \text{ (Fig. 22c).}
\]

Further examples:

\[
\ln 25 = 3.22; \ln 145 = 4.97;
\]
\[
\ln 0.04 = -\ln \left( \frac{1}{0.04} \right) = -\ln 25 = -3.22;
\]
\[
\ln 1.6 = 0.47; \ln 1.515 = 0.415;
\]
\[
\ln 0.66 = -\ln \left( \frac{1}{0.66} \right) = -\ln 1.515 = -0.415.
\]
Powers of any desired numbers

Powers of the form \( a^n \) are obtained by moving C 1 (or C 10) into position above (or below) the basic value \( a \) of the corresponding LL scale and moving the cursor to C-n. \( a^n \) can then be read from LL, e.g. \( 1.277^{22} = 1.72 \) (Fig. 23).

Place C 1 under LL\(_2\) 1-277 and find the result, 1.72, at C 2.22 on LL\(_2\).

Further examples:
- \( 3.752^{96} = 50; \quad 4.2^{216} = 22.3; \)
- \( 4.20^{216} = 1.364; \)
- \( 4.20^{216} = \frac{1}{4.20^{216}} = \frac{1}{1.364} = 0.734. \)

Fig. 23

Roots of any desired numbers

The root exponent is converted into a power exponent, on the principle \( \sqrt[n]{a} = a^\frac{1}{n} \), or else the Reciprocal Scale CI is used for the actual setting, e.g. —

\( \sqrt[4.4]{23} = 2.04; \) place CI-10 above LL\(_3\) 23 and find the result, 2.04, at CI-4.4 on LL\(_2\).

Further examples:
- \( \sqrt[0.6]{15.2} = 9.35 \) (CI 10 to be placed above LL\(_3\) 15.2; reading to be taken on LL\(_3\)).
- \( \sqrt[5]{2} = 1.149 \) (CI 1 to be set under LL\(_2\) 2, reading to be taken on LL\(_2\)).
\[ \frac{1}{20} = 1.82 \text{ (set CI 10 above LL, take reading on LL).} \]
\[ 5 \quad 1 \quad 1 \quad 1 \quad 1.149 = 0.871 \]
\[ 2 \quad 1 \quad 0.05^5 \]
\[ 5 \quad 1 \quad 0.05^5 \quad 1 \quad 1.82 \quad 0.55 \]

**Logarithms to any desired base**

Place the beginning of the C scale above the base on the LL scale; this provides a table of the relevant logarithms, e.g. —

\[
\begin{align*}
\log 5 &= 1; \quad \log 60 = 2.54; \quad \log 800 = 4.15. \\
\log 20 &= 1.301; \quad \log 2 = 0.301; \quad \log 800 = 2.9. \\
\log 200 &= 7.65; \quad \log 22 = 4.46; \quad \log 189 = 0.92.
\end{align*}
\]

**The additional scales on the “Electro” Slide Rules (No. 1/98, 4/98, 111/98)**

The Efficiency Scale \( W \) is provided on slide rules 1/98 and 4/98 in the “well” (on the body of the rule, below the slide), and on slide rule 111/98 on the lower body of the rule.

It interacts with scales A and B, this being the reason why the symbols kW and HP are shown at the right-hand end of the latter scales. The left-hand half of the W scale applies to D.C. generators, the right-hand half to electric motors.

**Efficiency of D.C. Generators.**

Examples:

(1) Calculate the efficiency of a D.C. generator of 134 HP and 80 kW.

We use the cursor line to place B 134 (for 134 HP) under A 80 (for 80 kW). The metal pointer of the slide (slide rules 1/98, 4/98) or index C1 (slide rule 111/98) then shows the value \( \eta = 80\% \) efficiency on the W scale.

(2) What power is obtained, at 30 HP, from a D.C. generator with an efficiency of 88\%?

We place the metal pointer of the slide (slide rules 1/98, 4/98) or, with the help of the cursor, the index C1 (slide rule 111/98) at 88\% of the W scale (left-hand half); the power (19.7 kW) can now be read on scale A, above B 3 (for 30 HP).
**Efficiency of Electric Motors.**

Examples:

(1) What is the efficiency of a motor supplying 20 HP at 17.1 kW?

Place B 2 (for 20 HP) under A 17.1 kW; at the metal pointer of the slide (models 1/98, 4/98) or underneath C 1, with the help of the cursor (model 111/98), we find the reading for \( \eta \), i.e. 87\%.

(2) What power is supplied by a motor with 500 V and 12 amps. (i.e. 6 kW) if the efficiency, \( \eta \), is 80\%?

Place the pointer (models 1/98, 4/98) or the index C 1, using the cursor (model 111/98) on the number 80 of the efficiency scale W (right-hand half); the answer, 6.45 HP, is then found on B, underneath A 60 (for 6 kW).

**Scale for Voltage Drop V**

The scale for the voltage drop, in the case of the slide rules 1/98 and 4/98, is provided in the well of the slide rule, underneath the slide, while in the case of slide rule 111/98 it appears on the lower body of the slide rule, below the efficiency scale. The voltage drop scale (marked in red) too interacts with scales A and B.

The voltage drop in a copper conductor for D.C. or A.C. with induction-free loading is calculated by the formula:

\[
e = \frac{I \times L}{c \times a}
\]

The factor \( c = 56 \) (conductivity of material) is already taken into account in the voltage drop scale. All that is necessary, therefore, is to multiply the current \( I \) (in amperes) by the total length \( L \) of the conductor (in yards or metres) and divide it by the cross section \( a \) of the conductor (in circ. mils or \( \text{mm}^2 \)). On Slide Rules 1/98 and 4/98 the voltage drop is then shown, in volts, by means of the pointer and on the red scale in the well of the slide, while on Slide Rule 111/98 the voltage drop is shown, by means of the cursor, underneath C 1 on the volt scale on the lower body of the slide rule.

Example:

Calculate the voltage drop in a copper conductor of a total length of 500 yards (outward and return lead) with a cross section of 41,000 circular mils, the current being 12.9 amps.

Place B 1 under A 12.9 (for 12.9 amps.), move the cursor to B 500 (for 500 yards), and move the slide in order to bring B 41 (for 41,000 circ. mil.) into position underneath the cursor mark. On Slide Rules 1/98 and 4/98 the voltage drop (4.81 V) is found against the pointer on the slide on the volt scale. In Slide Rule 111/98 the voltage drop is found, by the aid of the cursor, underneath C 1 on the volt scale, and the reading is here again 4.81 V.
The voltage drop scale only indicates the correct place for the decimal point if the values I, L and a can be set direct on the scales A and B, taking into account the initial figures at the left-hand end of the slide. If this is not possible, then ten times the value or one tenth thereof is set, and the result is then appropriately divided or multiplied, e.g. to calculate the voltage drop in a 4-km railway circuit, the trolley wire having a cross sectional area of 50 mm² and the current consumption being 29 amps., proceed as follows:

Place B 1 under A 2·9 (for 29 amps.), place the cursor on B 40 (for L = 400 m) and bring B 5 (for cross section of 50 mm²) into position underneath the cursor line.

The reading (4·13 V) is obtained on the voltage scale. For 4000 m, therefore, the voltage drop will be 41·3 V.

The mark 746 facilitates the formation of tables for conversion of HP to kW.

Place the 746 of B under A 1 or B 1, or place B 100 under 746 on A; this provides a table for kW and HP.

Marks for Calculation of Resistance and Weight.

Marks R and W for calculation of resistance and weight on ELECTRO SLIDE RULES Nos. 1/98, 4/98, 111/98.

The black mark R (on scale C) for copper, is used for calculating the ohmic resistance of conductors, (at 60° F.).

Example: What is the ohmic resistance of a copper conductor with a diameter of 0·036 inch (36 mils) and a length of 1,340 yards?

Use the cursor line to place 0·036 on the lower scale of the slide rule (D 36) and 1,340 yards on the upper scale of the slide B 1340 opposite each other. The resistance: 31·6 ohms, can then be read off on B, above the mark R on scale D. (Corresponds to the B.E.S.A. standard for annealed copper).

The black mark W (on scale C) for copper, is used for calculating the weight of the conductor.

Example: What is the weight of a copper conductor with a diameter of 0·018 inch (18 mils) and a length of 1,340 yards?

Using the cursor line, place 1,340 yards on the upper scale of the slide (B 1340) above the mark W (on D). The weight (3·94 lbs.) can then be read off on B over the diameter, 0·018", on the lower scale of the slide rule D.

The same setting can be used to find the weights of conductors of the same length but with different diameters, e.g. for 0·022" (5·90 lbs.), 0·025" (7·6 lbs.), etc.
CASTELL Precision Slide Rules

are the result of many years experience, a culmination of the skilled workmanship of men with long training. The rules are unsurpassed for precision and should be handled with care.

Slide Rules of Specialised Wood Construction

are impervious to climate; they should nevertheless be protected from any considerable temperature fluctuations and from humidity. The resilient base gives the Slide Rule great elasticity, making it easier to move the slide and adjust its mobility. There are metal inserts ensuring exceptional stability in the basic structure of the Slide Rule and preventing deformation by climatic influences.

Slide Rules of Geroplast

are absolutely "climate-proof", as well as heat-proof and damp-proof, and they stand up to the effects of the majority of chemicals. Geroplast Slide Rules should not, however, be allowed to come in contact with corrosive liquids or strong solvents, as these — even if the material itself remains unaffected — are at any rate liable to cause the colour of the graduation marks to deteriorate.

The following applies both to Wood Slide Rules and to Geroplast Rules:

To preserve the legibility of the graduations, the facial scales and the cursor should be protected from dust and scratches, and cleaned at frequent intervals with the special CASTELL cleaning agent No. 211 (liquid) or No. 212 (in paste form).

Never clean them with alcohol!